

Finite contracts that preserve incentives and funding

A certificate-preserving finite-support theorem

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Research showcase note 29 September 2026

Abstract

A venture-finance assessment may specify continuous payment lotteries and privately chosen financial positions. In a capped, pooled institution with a committed dealer quote and funded access, every feasible assessment has an equivalent representation on at most eleven joint outcomes, including at most nine service-payment outcomes. The reduction preserves the actual funding requirement, both privately optimized financial values, effort choice, dealer participation and the general partner's utility. Its economic content is preservation of the upper certificates against private deviations. The geometric step is the classical theorem of Carathéodory; its elementary proof and the required financial duality argument are included here.

1 Institution, timing and continuation

A general partner (GP) manages a venture at scale $n \in [0, 1]$. Conditional on financing, technical success has probability $p = 4/5$ under effort H and $p_0 = 2/5$ under effort L . Low effort gives private benefit bn , where $b = 1/5$. Successful output is $2n$ at each of two future dates; the input costs Kn , where $K = 167/100$. Household dated prices are β^d , with $\beta = 4/5$, and personal discount weights are δ^d , with $\delta = 1/2$.

Routing delivers matching probability $m(q) = cq/(q + \ell)$ at queue $q \geq 0$, with fixed $c, \ell > 0$. A fixed implemented confidential-registry policy has certificate probability ρ and expected testing cost C . Put

$$t = \rho m(q), \quad T(q) = Cm(q) + uq, \quad (1)$$

where $u \geq 0$ is the application value and $u(q + \ell)/c$ the owner's rent per match. Thus T pays both the actual test cost and the routing-implied rent. The original calibration has $c = 1/2$, $\ell = 1$, $0 \leq \rho \leq 3/8$, so $t < 1$.

The certificate is one pooled effort-information node. The registrar publishes only its Boolean certificate. Private test identities, instructions and results do not create extra effort cells. The certificate-contingent input is settled before effort. Service-payment marks and the independent public insurance mark resolve after effort, and the GP cannot make technical failure occur by vetoing the investment. Service compensation is unvested and unassignable. Quotes are committed before private orders: the dealer does not reprice after observing an effort-correlated order.

The branches are success S , failure F , and nonfinancing N . Their physical probabilities, conditional on a pure effort policy, are

$$P_H = (tp, t(1 - p), 1 - t), \quad P_L = (tp_0, t(1 - p_0), 1 - t). \quad (2)$$

If the GP uses H with probability $\vartheta \in [0, 1]$, the actual quote and LP output law are

$$Q = \vartheta P_H + (1 - \vartheta)P_L, \quad \bar{p} = \vartheta p + (1 - \vartheta)p_0, \quad \bar{F} = 2\bar{p}(\beta + \beta^2). \quad (3)$$

The private effort randomization is independent of the service lottery. On S , the common service vector $Z = (Z_1, Z_2)$ has any probability law μ on $[0, 2n]^2$. On F, N , $Z = (0, 0)$; denote these branch laws by μ_e .

Purchased claims deliver first-date financial wealth $Y_a \in [0, 4]$ under effort policy a . This includes the first-date liquidation value of assignable purchased claims at both dates. The unassignable service claim Z_2 remains separately dated. The bound four is the actually backed aggregate payout limit, with the stated funded access and reserve technology available for every permitted private policy. A mandatory dealer payment $A \geq 0$ is prepaid from primary finance; $f \geq 0$ is the GP's remaining disposable current cash. Dealer participation concerns its whole bundle of securities. Dated reserves and collateral remain required; an expected-profit inequality is not a replacement for them.

Actual successor value. For each branch, $\mathcal{V}_e$ is the actual optimized value at the successor's eligibility state. It includes funded dealer access, privately mixed effort, implemented confidential-registry policies and directed posting when eligible; an ineligible successor consumes wealth, so $\mathcal{V}_e(W) = W$. On the relevant interval $W \in [0, 6]$, the continuation is continuous and satisfies

$$|\mathcal{V}_e(W) - \mathcal{V}_e(W')| \leq L|W - W'|, \quad L = 200/3. \quad (4)$$

This is the funded-access continuation regularity condition used by the selected theorem. A precise construction and the source of its bound are provided in Appendix A. A fixed-pure-high continuation with a rich-wealth refusal formula is not substituted for $\mathcal{V}_e$.

2 Feasible assessments and their private financial problems

For each $a \in \{H, L\}$ and branch e , let $\gamma_{a,e}$ be a joint law of (Z, Y_a) with service marginal μ_e and $0 \leq Y_a \leq 4$. Policies may differ across the two privately chosen efforts. Each policy is purchased in advance as a contingent schedule; the public post-effort mark implements its randomization conditional on the service mark. It is not a new unfunded purchase after the realized payment is observed.

Write its premium and selected utility as

$$\mathcal{P}_a = \beta \sum_e Q_e \mathbb{E}_{\gamma_{a,e}} Y_a, \quad (5)$$

$$h_a = f - \mathcal{P}_a + \sum_e P_{a,e} \mathbb{E}_{\gamma_{a,e}} [\delta \mathcal{V}_e(Z_1 + Y_a) + \delta^2 Z_2] + \mathbf{1}_{\{a=L\}} t b n. \quad (6)$$

The financial problem for effort a maximizes (6) over all such joint laws with $\mathcal{P}_a \leq f$. In particular the quote Q stays fixed when effort privately deviates. A feasible assessment requires selected policies that attain both maxima, and the following rows:

$$f + A + T(q) + \sum_e Q_e \mathbb{E}_{\mu_e} [\beta Z_1 + \beta^2 Z_2] \leq t(\bar{F} - K)n, \quad (7)$$

$$\mathcal{P}_a \leq f \quad (a = H, L), \quad (8)$$

$$\begin{aligned} & A + \vartheta(\mathcal{P}_H - \beta \sum_e P_{H,e} \mathbb{E} Y_H) \\ & + (1 - \vartheta)(\mathcal{P}_L - \beta \sum_e P_{L,e} \mathbb{E} Y_L) \geq 0, \end{aligned} \quad (9)$$

$$\vartheta(h_L - h_H) \leq 0, \quad (1 - \vartheta)(h_H - h_L) \leq 0. \quad (10)$$

The source row uses actual mixed-effort output. The payment A is charged once in this row and received once in dealer participation. The last row is actual effort optimality: both efforts tie in an interior mix, and a pure selection satisfies the appropriate weak inequality. If entry costs h are part of the fixed outer parameters, GP net utility is $\vartheta h_H + (1 - \vartheta)h_L - h$.

All sums below run over nonempty branches, so $Q_e > 0$. From (2)–(3), these branches also have $P_{a,e} > 0$ and

$$P_{a,e}/Q_e \leq 3. \quad (11)$$

Indeed the success ratios are $p_a/\bar{p} \leq 2$, failure ratios are $(1 - p_a)/(1 - \bar{p}) \leq 3$, and the nonfinancing ratio is one. Empty branches are simply omitted.

3 Attained certificates for every financial deviation

For $a \in \{H, L\}$ define

$$\Psi_{a,e}(z; \lambda) = \max_{0 \leq y \leq 4} \left\{ \delta \mathcal{V}_e(z + y) - \lambda \beta \frac{Q_e}{P_{a,e}} y \right\}, \quad (12)$$

$$d_a(\lambda) = \lambda f + \mathbf{1}_{\{a=L\}} tbn + \sum_e P_{a,e} \mathbb{E}_{\mu_e} [\delta^2 Z_2 + \Psi_{a,e}(Z_1; \lambda)]. \quad (13)$$

Each maximum is attained and $\Psi_{a,e}$ is continuous in z , by compactness and continuity.

Lemma 1 (Exact private-finance certificate). *For any fixed service law and outer parameters, the effort- a financial problem has an attained maximum h_a^* , and*

$$h_a^* = \min_{1 \leq \lambda \leq 125} d_a(\lambda).$$

The minimum is attained, including at $f = 0$. An affordable selected policy is a financial best response if and only if $h_a = d_a(\lambda_a)$ for some $\lambda_a \in [1, 125]$.

Proof. Probability laws on the compact service–payoff spaces with the fixed service marginals form a compact convex set for weak convergence. Premium and payoff are continuous linear functionals. The affordable set is nonempty (choose $Y = 0$) and compact, so a maximum exists. For any affordable law and $\lambda \geq 1$, write the objective as

$$f - \mathcal{P}_a + G_a = \lambda f + (G_a - \lambda \mathcal{P}_a) - (\lambda - 1)(f - \mathcal{P}_a),$$

where G_a includes future utility and the low-effort benefit. The last term is nonpositive. Maximizing the remaining integrand conditional on (e, Z) gives (13). This proves weak duality. A measurable maximizer exists since the objective is continuous on a compact interval, so integrating the pointwise maximum equals the supremum over all admissible conditional laws.

When $f > 0$, the premium-zero law is strictly affordable. The compact convex image of laws under (premium, future payoff) has a supporting line at the optimal affordable point. Equivalently, separation of this image from strictly larger affordable objective values supplies a nonnegative multiplier for the premium constraint. Adding the unit cost of the cash used gives $\lambda \geq 1$ and equality with $d_a(\lambda)$. This is strong duality; no concavity of $\mathcal{V}_e$ is needed because the optimization variable is a probability law.

The Lipschitz bound (4) implies, for $y \geq 0$,

$$\delta \mathcal{V}_e(z + y) - \lambda \beta (Q_e/P_{a,e})y \leq \delta \mathcal{V}_e(z) + [\delta L - \lambda \beta (Q_e/P_{a,e})]y.$$

Thus $y = 0$ maximizes every envelope whenever

$$\lambda \geq \frac{\delta L}{\beta} \max_e \frac{P_{a,e}}{Q_e} \leq \frac{1/2 \cdot 200}{4/5 \cdot 3} 3 = 125.$$

Above 125, the dual equals a constant plus λf , so increasing λ cannot improve it. Continuity on $[1, 125]$ proves attainment there. If $f = 0$, nonnegative payoffs and positive Q_e force $Y = 0$ almost surely; $\lambda = 125$ makes the same zero-payoff choice maximize every envelope and directly proves equality. The last assertion follows from weak duality and attained equality. $\square$

4 Finite representation

Theorem 2 (Finite contracts with actual mixed effort and funded access). *Every feasible assessment in Sections 1–2 has a representation on at most eleven joint atoms across S, F, N , preserving its current source, both privately optimized financial values, actual effort mixture, dealer participation and GP utility. Its success-conditional service law has at most nine atoms. More generally, with k nonempty branches and one service-paying branch, the corresponding bounds are $k + 8$ joint atoms and nine service atoms.*

Lemma 3 (Classical finite convex representation). *If X is compact and $F : X \rightarrow \mathbb{R}^d$ is continuous, the average $\int F d\eta$ under any probability law η can be written as a convex combination of at most $d + 1$ values of F .*

Proof. First, a finite convex combination with more than $d + 1$ positively weighted points has affinely dependent points: there are numbers a_j , not all zero, with $\sum_j a_j = 0$ and $\sum_j a_j F(x_j) = 0$. Some $a_j > 0$. Set $s = \min_{a_j > 0} w_j / a_j$ and replace w_j by $w_j - sa_j$. The average and total weight do not change, all weights remain nonnegative, and at least one becomes zero. Iteration leaves at most $d + 1$ points. This is Carathéodory's theorem.

The convex hull of $F(X)$ is therefore the image of the compact product of $d + 1$ copies of $F(X)$ and the probability simplex under the weighted-sum map. It is compact, hence closed. Approximate the integral of the continuous bounded map F by finite convex combinations (partition its compact image into sets of arbitrarily small diameter). Their limit is the integral and lies in the same convex hull, proving the claim. $\square$

Proof of Theorem 2. Fix all outer parameters, including $A, q, n, (\rho, C), f$ and ϑ . For each effort choose the selected attaining policy and a multiplier λ_a from Lemma 1. Conditional on branch and the common service draw, couple the two financial policies. For example, take the product of their regular conditional laws. Weight branches by Q_e . This produces a probability law η on the compact disjoint union of spaces

$$\begin{aligned} x &= (e, z_1, z_2, y_H, y_L), & y_H, y_L &\in [0, 4], \\ (z_1, z_2) &\in [0, 2n]^2 \ (e = S), & z_1 = z_2 = 0 \ (e \neq S). \end{aligned}$$

Only the portfolio corresponding to the privately selected effort is purchased. The mathematical coupling never awards both portfolios.

Preserve $k - 1$ branch-indicator expectations and the following eight scalar expectations under η :

1. the source cost $s(x) = \beta z_1 + \beta^2 z_2$;
2. the two premium integrands $b_a(x) = \beta y_a$;
3. the two physical future-utility integrands

$$v_a(x) = \frac{P_{a,e}}{Q_e} [\delta \mathcal{V}_e(z_1 + y_a) + \delta^2 z_2];$$

4. the two dual-certificate integrands

$$d_a^0(x) = \frac{P_{a,e}}{Q_e} [\delta^2 z_2 + \Psi_{a,e}(z_1; \lambda_a)];$$

5. the dealer-profit integrand

$$g(x) = \beta \left[\vartheta \left(1 - \frac{P_{H,e}}{Q_e} \right) y_H + (1 - \vartheta) \left(1 - \frac{P_{L,e}}{Q_e} \right) y_L \right].$$

This is a continuous map into $\mathbb{R}^{k+7}$. Lemma 3 replaces η by a law η' with the identical vector and at most $k + 8$ points. Normalization and the first $k - 1$ indicators preserve all k branch masses. The success marginal of η' becomes the new service law μ' ; both effort-contingent financial policies have that same marginal.

The preserved source cost and fixed outer parameters keep (7) unchanged. The two preserved premiums keep affordability unchanged. The two preserved physical values keep h_H, h_L unchanged. The two preserved dual integrals keep $d_H(\lambda_H), d_L(\lambda_L)$ unchanged, despite the change in the service marginal. Their original equalities $h_a = d_a(\lambda_a)$ therefore survive. Weak duality bounds every permitted new private financial policy above by this preserved certificate, and the compressed selected policy attains it. Consequently both compressed policies are genuine best responses.

The preserved g and fixed A keep actual dealer participation unchanged. The fixed ϑ and preserved h_H, h_L keep effort optimality and GP utility unchanged. Physical caps hold at every retained point, and the stipulated backing and post-effort mark implement the new laws. No optimality of the original outer contract was used.

For three branches, $k + 8 = 11$. Each of the $k - 1$ nonservice branches with positive mass requires at least one atom; at most $(k + 8) - (k - 1) = 9$ atoms remain on the service-paying branch. Its marginal can have still fewer distinct service points. If that branch is empty there is no nonzero service payment and the bound is immediate. $\square$

5 Interpretation and boundaries

The reduction preserves optimized behavior, not merely average transfers. A changed service lottery can alter which privately purchased portfolio is attractive. Preserving both achieved utilities and their attained upper certificates rules out exactly that failure. Ten moments suffice in the three-branch case: two branch masses plus the eight economic quantities above.

The result concerns every feasible assessment, so it does not rely on a maximizing outer contract. *At most eleven* is an upper bound, not a minimality statement. Nine refers to the service law conditional on the service-paying branch; the nonservice branches pay zero. Additional observable pre-effort cells, order-by-order repricing, different financial caps, or security-by-security dealer participation require their own constraints and are not included by implication.

At a zero-probability boundary, remove empty branches before applying the proof. For this technology, the structural success and failure likelihood ratios remain bounded as t vanishes. There is no need to evaluate a $0/0$ expression. The theorem is an exact local representation and does not itself impose aggregate market clearing.

A The actual funded-access continuation

This appendix specifies the economic continuation behind (4). The finite-support proof is conditional on that actual continuation and its displayed regularity; it does not optimize a different successor institution.

Terminal funded access. Let $U_A(W)$ be the optimal value of one resolved viable terminal opportunity, including refusal W . For a forecast success probability $\pi \in [p_0, p]$, define $\alpha = (\pi - p_0)/(p - p_0)$. Choose $n \in [0, 1]$, service $0 \leq z_d \leq 2n$, discretionary cash $B \geq 0$, mandatory access payment $A \geq 0$, and two optimal financial policies under the common quote $(\pi, 1 - \pi)$. Each policy selects purchased first-date claims in $[0, 4]$, with premium at most B . Their exact effort- a utility is

$$H_a = B - \beta \mathbb{E}_{(\pi, 1-\pi)} Y_a + p_a(\delta z_1 + \delta^2 z_2) + \delta \mathbb{E}_{(p_a, 1-p_a)} Y_a + \mathbf{1}_{\{a=L\}} bn.$$

Impose $\alpha(H_L - H_H) \leq 0$, $(1 - \alpha)(H_H - H_L) \leq 0$, aggregate dealer participation as in (9), and

$$\begin{aligned} B + A + \pi(\beta z_1 + \beta^2 z_2) &\leq W + [2\pi(\beta + \beta^2) - K]n, \\ B + A &\geq W - Kn. \end{aligned}$$

The second row is nonnegative LP primary capital. Maximize $\alpha H_H + (1 - \alpha)H_L$. Extra mandatory payments are permitted: freeing them as discretionary cash would change incentives. This defines U_A through its full financial optimization and funded budget rows.

Why the common bound is 200/3. First-date maximal payoff/price ratios, including consumption, are

$$R_H(\pi) = \max\{1, 1/(2\pi)\}, \quad R_L(\pi) = \max\{1, 3/[8(1 - \pi)]\}.$$

Later claims have 5/8 times the corresponding first-date ratio, so moving a later claim to date one with factor β preserves its price and reduces aggregate cap use. Both efforts consume cash for $\pi \in [1/2, 5/8]$; outside this interval the profitable hedge is respectively success or failure. Write $T_z = \delta z_1 + \delta^2 z_2$ and $Q_0 = 2\beta + 2\beta^2 = 72/25$. The cheapest service satisfies $\beta z_1 + \beta^2 z_2 \geq (8/5)T_z$. For $\pi < 1/2$, mixing equates utility to low's cash-policy value $B + p_0 T_z + bn$. Funding gives

$$U - W \leq (2.88\pi - 1.47)n + (.4 - 1.6\pi)T_z - A \leq -.03n.$$

Here $\pi \in [.4, .5)$, so both the first coefficient (at most $-.03$) and the second are nonpositive. These mixtures do not beat refusal. Pure low has gain bounded by $(p_0 Q_0 - K + b)n = -159n/500$.

For $\pi > 5/8$ and a binding failure payoff cap four, let $g_L = 3.2\pi - 2$. Indifference gives $T_z = n/2 + (5/2)g_L$. Since $T_z \leq 3n/2$, it follows that $\pi \leq 3/4$. Actual dealer participation gives $A \geq 8(\pi - .4)(.8 - \pi)$. Therefore

$$\begin{aligned} U - W &\leq (2.08\pi - 1.27)n - (4\pi - 2)g_L - A \\ &\leq -\frac{24}{5}\pi^2 + \frac{172}{25}\pi - \frac{271}{100} \leq -\frac{367}{1500}. \end{aligned}$$

The quadratic's maximum occurs at $\pi = 43/60$. Pure high cannot have a binding four-unit low-effort deviation: the deviation gains $14/25$ over cash, exceeding the maximal service incentive margin $2n/5$. At the cash/insurance tie boundaries one may select cash or an uncapped indifferent hedge. Thus a positive-gain optimum has $\pi \geq 1/2$ and only uncapped profitable hedges or no trade.

Put $dT = T_z - n/2 \geq 0$ and $\kappa_\pi = K - (52/25)\pi \geq 3/500$. High consumes B , and actual funding gives

$$W \geq B + A + \kappa_\pi n + \frac{8\pi}{5}dT, \quad U_A = B + \frac{2}{5}n + \frac{4}{5}dT.$$

Since $8\pi/5 \geq 4/5$, this implies

$$W \geq \frac{3}{500}n, \quad U_A - W \leq \left(\frac{2}{5} - \kappa_\pi\right)n - A \leq \frac{197}{500}n, \quad W \leq U_A(W) \leq \frac{200}{3}W.$$

The feasible auxiliary graph is compact on bounded wealth intervals: the source bounds $B + A$, $z_d \leq 2$, $Y_a \leq 4$, and $\pi \in [p_0, p]$. The effort-specific financial value is continuous (a finite linear claim problem with continuous positive prices); its optimal-policy graph is closed. Zero-scale technical labels cannot improve refusal, since consolidated participation bounds the source gain by $(317/500)n$. A positive-gain maximizing sequence stays away from zero scale, so its limit is an actual contract; otherwise refusal attains the value.

Increasing wealth by s and the mandatory payment by s preserves the old financial policy, primary rows and incentives. Hence U_A is nondecreasing. For $0 < W_1 < W_2$, scale every control of a positive-gain W_2 optimum by W_1/W_2 . The budget and incentive rows are homogeneous; an uncapped profitable hedge optimally scales and stays below four. Thus $U_A(W_1) \geq (W_1/W_2)U_A(W_2)$. Refusal has the same property. The source bound gives $U_A(0) = 0$, and

$$0 \leq U_A(W_2) - U_A(W_1) \leq \frac{W_2 - W_1}{W_2} U_A(W_2) \leq \frac{200}{3} (W_2 - W_1).$$

Confidential registry and posting. After routing, evidence owners share a clue with probabilities $1/2, 1/2$ and viability posteriors $2/5, 3/5$. Each good-only test detects viability with probability $1/2$; their conditional outcomes are independent. Founder and incumbent testing cost $1/1000$ and $1/2000$. At each clue, the effective choices are no test, incumbent only, or both. Their contributions to (ρ, C) are

	none	incumbent	both
L	$(0, 0)$	$(1/10, 1/4000)$	$(3/20, 3/4000)$
H	$(0, 0)$	$(3/20, 1/4000)$	$(9/40, 3/4000)$

A deterministic policy adds one entry from each row. Founder-only duplicates are more costly than incumbent-only; monotonicity of U_A makes them unnecessary. Private randomized instructions generate the convex hull of these nine points. Its lower boundary is

$$C_{\min}(\rho) = \begin{cases} \rho/600, & 0 \leq \rho \leq 3/20, \\ \rho/400 - 1/8000, & 3/20 \leq \rho \leq 1/4, \\ \rho/150 - 7/6000, & 1/4 \leq \rho \leq 13/40, \\ \rho/100 - 9/4000, & 13/40 \leq \rho \leq 3/8. \end{cases}$$

Indeed each deterministic policy is on or above all four affine lines, and adjacent vertices $(0, 0)$, $(3/20, 1/4000)$, $(1/4, 1/2000)$, $(13/40, 1/1000)$, $(3/8, 3/2000)$ are actual policies. Randomizing adjacent policies gives equality throughout each segment.

The registrar records genuine performance, reimburses actual costs and pays the promised net matching rent from prefunded reserves. The Boolean certificate cannot be suppressed. Unused reserves return to their funder without an early information leak. Complying evidence owners receive the same net rent after any clue report; skipping does not improve utility and forfeits any compliance rent, while an uninstructed extra test is unreimbursed. Truthful reporting and prescribed testing are therefore weakly implemented.

An eligible successor with wealth W has the actual value

$$V(W) = \max \left\{ W, \sup_{\substack{q>0, 0<\rho\leq 3/8 \\ W\geq T}} \left[tU_A \left(\frac{W-T}{t} \right) - h \right] \right\}, \quad t = \rho m(q), \quad T = C_{\min}(\rho)m(q) + uq.$$

This normalization divides actual certificate-contingent source rows, dealer loss and financial premium by t . It does not divide the physical financial payoff cap: a claim purchased for

unconditional budget B_0 has failure payout $\min\{4, B_0/[t\beta(1 - \pi)]\}$, exactly the conditional problem with $B = B_0/t$. Actual test/rent premiums, stipends and unused refunds retain their distinct funded ledgers.

For a fixed option (q, ρ) , the extension $\max\{W, tU_A((W - T)_+/t) - h\}$ is 200/3-Lipschitz, including $W < T$ where it equals refusal. Taking the supremum preserves this bound. At $t = 0$, $0 \leq U_A(x) - x \leq 197/500$ gives the continuous limiting value, which does not exceed refusal. Ineligible branches have $V(W) = W$. These are the successor values used in the finite-support theorem.

Source and version note

This standalone presentation is drawn from Elies Lucio and Hugues Pouget, *Information, Venture Finance, and International Portfolios*, integrated research manuscript (title-page date 9 September 2026), specifically the theorem *Finite representation with actual mixed effort and funded access* and its programme in `mixed_quote_newborn.tex`. Its financial dual develops the compact-law argument in `newborn_finite_support.tex`; the funded-access and registry continuation comes from `mixed_effort_prices.tex` and `registry_information_choice.tex`. The geometric result in Lemma 3 is classical Carathéodory convex geometry. The earlier seven-atom, fixed-high-quote theorem has a narrower domain and is not asserted here. All definitions and arguments needed for the finite-support proof are included; no inaccessible local-file reference is required.