MANUSCRIPT EXTRACT

Finite representation of application demand

T1 technical supplement

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Sections 29.1-29.3; original manuscript pages 163-165.

The original eleven-atom representation remains valid for its stated value and incentive objects. Preserving the additional statistic of complete lifetime applications gives a representation with at most twelve joint newborn atoms and twenty-four actual successor atoms.

The extract contains Proposition 29.1, its complete proof, the induced aggregate application correspondence, and the explicit boundary: fixed-rent compactness alone does not establish the general market-clearing crossing.

Domain: the capped, pooled, funded-access confidential-registry institution, with actually optimal senior responses and population mixtures of complete optimal careers. This is an extension of the preserved statistic, not a replacement of eleven by twelve everywhere.

How to read this supplement

This is a manuscript extract, not a self-contained proof note. The mathematical text, theorem numbers and printed cross-references are retained. References to sections and bibliography entries outside the extract point to the complete manuscript. Boundary-page content outside the selected sections is visually trimmed. The relevant theorem statements and their full local proofs are retained.

29.1 Actual careers as the routing rent varies

Keep the actual delayed-result confidential registry, financed output $(2, 2)$, funded mixed-effort value U_A, and senior entry cost $h_s = 1/100000$. For every $u > 0$ define the true senior value

$$V_u(W) = \max\left\{W, \max_{\substack{q>0,\rho>0\\ W\geq T}} tU_A\Big(\frac{W-T}{t}\Big) - h_s\right\}, \qquad t = \frac{\rho c q}{q+\ell}, \quad T = \frac{C_{\min}(\rho)cq}{q+\ell} + uq.$$

On compact positive-u intervals and $W \in [0, 6]$, this is jointly continuous. It has wealth Lipschitz bound $L = 200/3$. For any common bound on relevant queues, changing u changes T by $q\,du$, giving a local price Lipschitz bound $L\overline{q}$. The inactive pasting is continuous because at $W = T$ the launch payoff is below refusal. Taking maxima preserves these bounds. The value is nonincreasing in u: a lower owner premium can be replaced by the same increase in actual mandatory access, preserving disposable cash, claims and financial incentives while weakly relaxing dealer participation. This uses the proved funded institution.

Let $G(u)$ be the maximum *gross* newborn value in the complete MQ program using this actual V_u, before the newborn's nonpecuniary activation cost. Both optimized financial policies, actual mixed effort, dealer participation and the full registry frontier remain in the program. For fixed u it is attained. Its definition is independent of the newborn activation cost, which is additive and sunk after entry. On compact positive-u intervals the feasible graph is closed in a common compact action space and the objective is continuous. Thus G is upper semicontinuous. This fact alone does not give a closed graph for its maximizers.

29.2 Exact optimal application demand

For one entered newborn and its actual senior policies, lifetime applications are

$$a = q_y + \int q_s(W)\,\nu(dW, dZ_2, da, de). \tag{282}$$

The measure ν uses the actual high/low financial orders and the actual effort mixture, as in (264).

Proposition 29.1 (Twelve-atom representation of full application demand)**.** *At fixed u with $G(u) > 0$, the complete population-convexified application set from gross-maximizing entering careers is a compact interval $[a_{\min}(u), a_{\max}(u)]$, with attained endpoints. Its exact finite representation requires at most twelve joint newborn atoms and twenty-four actual successor atoms. Eleven and twenty-two still suffice to select one value-maximizing career when its application statistic need not also be preserved.*

Proof. Begin with an arbitrary actually optimal service and financial law. At each service/financial support point attach a genuinely optimal senior action for both $W_H = Z_1 + Y_H$ and $W_L = Z_1 + Y_L$. The senior actions all give the same respective values $V_u(W_a)$, so their tie choices do not change either current financial best response. To the ten moments preserved in Theorem 27.1, add

$$\vartheta\frac{P_{H,e}}{Q_e}q_{s,H} + (1-\vartheta)\frac{P_{L,e}}{Q_e}q_{s,L}. \tag{283}$$

This is the actual senior-application contribution. All functions are continuous on the expanded compact set of service/financial points and optimal senior actions. The structural probability ratios are bounded; only positive-mass branches are included. Carathéodory's theorem now gives twelve support points, preserving all eleven moments. Both attained private dual certificates are preserved, so financial choices remain true best responses. Gross value remains $G(u)$, while (283) preserves (282) exactly.

The expanded finite parameter set is compact: selected senior actions lie in their compact maximizing graph; active controls have the uniform $tng \geq h_s$ bound, and refusal is a separate feasible action. Its application functional is continuous, so its extrema are attained. Population shares of GPs can use different complete optimal careers, giving their convex hull. This is a mixture across identities, not an unauthorized early lottery within a single pooled contract. If the same wealth arises from different histories, disintegrating the joint wealth/action law gives a Borel mixed policy at that wealth supported on optimal actions. All off-path values retain their Borel completion. □

Let potential GP birth mass be $\varepsilon > 0$ and newborn activation cost $h_y > 0$. Nonentering newborns have no financial wealth, unvested service or later transferable income; $V_u(0) = 0$ makes their later refusal optimal. The exact total application correspondence is

$$D(u) = \begin{cases} \varepsilon[a_{\min}(u), a_{\max}(u)], & G(u) > h_y, \\ \varepsilon[0, a_{\max}(u)], & G(u) = h_y, \\ \{0\}, & G(u) < h_y. \end{cases} \tag{284}$$

At equality, entry and refusal are both best responses. Every intermediate mass is achieved by their private entry mixture and, if needed, population mixtures of optimal entering assessments. All identities still age; the successor law becomes $\alpha\nu + (1-\alpha)\delta_{(0,0)}$ rather than deleting the nonentrants. The source bounds give

$$0 \leq a_{\max}(u) \leq 2 \max\left\{ \frac{c(3/8)a_0}{u} - \ell, 0 \right\}.$$

29.3 The unresolved general crossing step

For a fixed active founder population F, clearing requires $F \in D(u)$ at a rent covering its reservation costs. If D has nonempty compact interval values and a closed graph on $[u_-, u_+]$, with every element of $D(u_-)$ above F and every element of $D(u_+)$ below it, a crossing exists. Indeed, absent a crossing, the sets where all demand is above or below F are disjoint relatively open sets covering the connected price interval. Closed graph and boundedness make them open, a contradiction. No single continuous policy is needed for this argument.

The missing premise cannot be inferred from the fixed-u results. An exact counterexample to that inference has parameter $x \in [-1, 1]$, action $a \in [0, 1]$, objective $1 + a$ and constraint $xa = 0$. The feasible graph is closed and compact, and $a = 0$ is a uniformly feasible strictly positive benchmark. For $x \neq 0$ the unique optimum has value one; at zero it has $a = 1$ and value two. The value is upper semicontinuous but discontinuous, and the maximizing graph is not closed. This is a counterexample to the proposed mathematical inference, not an asserted

instance of an actual MQ contract.

In MQ, binding optimized incentive, source and dealer rows can lose nearby feasibility when the true V_u changes. A strict benchmark at another menu does not repair such a superior boundary assessment. Publicly mixing menus before effort leaves the invalid branch invalid; mixing their claims afterward need not preserve the two actual financial responses or funding. No general repair is presumed.

There is an actual high-rent inactive bound. With $S^* = \max_\rho\{\rho g - C_{\min}(\rho)\} = 117/800$,

$$J_u(W) + h_s \le \max_{q\ge 0}\{m(q)S^* - uq\} = \left(\sqrt{cS^*} - \sqrt{u\ell}\right)_+^2 .$$

When this is at most h_s, all senior values equal wealth. The newborn source inequality (268) then rules out positive financed scale. This is an off-equilibrium zero-demand bound, not a positive-rent empty-market allocation. It supplies neither the missing general maximizing graph nor the needed opposite endpoint for the original $h_y = h_s$ fixed-population question.