

Information can sustain venture finance across the economy

Theorem and complete rigorous proof

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Research showcase note, 29 September 2026

Abstract

A bounded stationary economy connects voluntary evidence, privately optimized venture finance, directed applications and household fund prices. It supports active careers with positive financing after actual success. Removing tests and every new viability signal makes every stationary real allocation in the same bounded candidate class inactive, even when household prices adjust. Funded off-menu continuations construct the inactive equilibrium. In a stated international sign region, mean trade balance changes from deficit to surplus while statewise net foreign assets remain unchanged.

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Reader's guide

This note presents the evidence-enabled common economy and its no-new-evidence comparison from the Integral Economics programme of Elies Lucio and Hugues Pouget. The theorem is constructive: firms, actual funded claims, household consumption, prices and international accounts belong to the same stationary economy. The comparison removes tests and every new viability signal, and solves household prices again. It is a theoretical mechanism, rather than an empirical estimate.

Section 1 gives the exact theorem and its proof. Appendix A defines the finite owner game, funded treasury and career clock. Appendices B–I supply all the optimization, source, entry and quantitative arguments used in the proof. Every cross-reference is internal to this PDF; the editable source is one file.

Economic notation. A GP is the venture financier/manager choosing a contract and then private effort; a primary LP supplies outside investment; a committed dealer supplies permitted private financial claims. The founder and incumbent are the two retiring evidence owners. They choose tests, authentic publication, reports and consent. Country households hold national funds containing the basic producers and these actual financing businesses. $n \in [0, 1]$ is real venture scale, $W \in [0, 6]$ is actual spendable GP wealth, q the applicant queue per venue, and $m(q) = cq/(q + \ell)$ its fill probability. Matching capacity is a maintained, conserved physical venue endowment; cash backing is provided separately by named dated basic-output reserves.

Finite lives and beliefs. Each GP is born with zero wealth, posts when young at age zero, and can post once more at age one. First payments from the young venture and liquidation of assignable old claims finance the senior choice. The original young venture pays again at age two; the senior venture pays at ages two and three. The GP then exits with no estate. One original technical success mark governs both productive payments of a venture. Ownership transfers never change an issuer, original maturity or reserve. Newborns receive no intergenerational grant. Information arrives after routing; the pretest clue has equiprobable viability posteriors $2/5$ and $3/5$. Two conditionally independent good-only tests have sensitivity $1/2$ and nonpecuniary costs $1/1000$ and $1/2000$.

Candidate class and preferences. The bounded candidate class fixes the finite owner/history template, twenty settlement stages, twenty possible directed transfer rows per stage and a fifty-unit cap per row, the current GP budget cap fifty, wealth cap six, aggregate remaining purchased-payment cap four and technical cap $4n$. It permits complete prepaid current claims, arbitrary admissible post-effort mark laws and all history-dependent private financial and effort deviations. GP utility is $\sum_j \delta^j C_{t+j}/A_{t+j}$ plus the low-effort benefit bn and less nonpecuniary listing cost h . Retiring owners value current cash divided by A_t , less their nonpecuniary test and creation costs. Households have $\mathbb{E} \sum_{t \geq 1} \beta^{t-1} \log C_{c,t}$, identical convex trading opportunities and initial half ownership of both funds after date-zero distributions and consumption have settled. There is no additional unindexed safe-goods asset.

1 General aggregate risk in the public nonlinear common economy

The common construction does not require two aggregate states or independent productivity over time. The relevant restriction concerns the conditional mean of the country-specific output shock. Under that restriction, the full dated contract domain has the same normalized optimization problem. Actual international signs then depend on a productivity–country-shock moment. The conditional mean-zero restriction is imposed at every public state.

The only household interdate securities are the two actual national funds. The fund-quantity uniqueness statement concerns this security menu.

1.1 Aggregate primitives and complete dated information

Retain the final public balanced-treasury institution of Section F. Owners have their actual finite testing, publication, report and consent histories. Their nonnegative treasury-paid cash settles privately after effort and supplies no additional GP/dealer signal. Primary input is irrevocably funded before technical orders. Every original current bill has actual named backing; the treasury is evaluated at the queue induced by the owner’s actual payoff. Complete conditional GP books remain optimal at every owner-null history. Every GP signal reaches its committed dealer, and no discretionary GP transfer creates a new owner-visible message.

Current retiring owners receive no reward indexed on an unrevealed future aggregate event. Old claims cannot refer to a future owner’s strategy, assignment, or the GP’s endogenous future menu or effort. The existing arbitrary post-effort mark laws and their true revelation dates remain available. Public macro history may index a current menu, but does not create new within-date owner stages or reports.

The numerical primitives remain

$$p = \beta = \frac{4}{5}, \quad p_0 = \frac{2}{5}, \quad \delta = \frac{1}{2}, \quad b = \frac{1}{5}, \quad K = \frac{167}{100}, \quad c = \frac{1}{2}, \quad \ell = 1, \quad h = \frac{1}{100000}.$$

A normalized payment x at date t means $A_t x$ actual goods. This convention applies to input, current cash, maintenance, purchased claims and their dated basic collateral. GP and retiring-owner cash utility remains the specified C/A utility; households retain ordinary log utility. No additional unindexed safe-goods asset is added. At each of its two original productive dates, one successful venture pays $A_t(2 - d_t)n$ in U and $A_t(2 + d_t)n$ in E . Its original technical success governs both dates. The uniform unassignable service covenant is

$$0 \leq z_j \leq \frac{7}{4}n, \quad j = 1, 2. \quad (1)$$

Purchased claims retain the aggregate remaining-payment cap four, with technical components additionally bounded by $4n$. There is no technical event at $n = 0$. Selling an old claim changes its holder, not its issuer, maturity, or original reserve.

Assumption 1.1 (An indexed conditional-mean-zero macro process). *Let Z_t be an exogenous stationary Markov process on a standard Borel space, with transition kernel P and specified invariant probability π . Measurable $A(Z)$ and $d(Z)$ satisfy*

$$0 < \underline{A} \leq A(Z) \leq \bar{A} < \infty, \quad |d(Z)| \leq d_{\max} \leq \frac{1}{4}, \quad Pd(z) = 0 \quad (2)$$

at every admissible public state z . Conditional on the current public state, future macro innovations are independent of all viability, test, technical-success, private-action and contractual randomizations, including their consistent off-path laws.

The invariant macro law is an exogenous primitive, not an assigned stationary wealth distribution. Iteration of its kernel gives

$$P^j d = 0 \quad (j \geq 1), \quad \int d d\pi = 0. \quad (3)$$

Thus productive payments have conditional mean two at both original dates. Joint iid (A, d) with $\mathbb{E}d = 0$ is a special case. Persistent A and state-dependent conditional distributions of d are allowed.

Lemma 1.2 (Full public dated-book equivalence). *Under Assumption 1.1, at world household resources $A_t \bar{H}$ with $\bar{H} > 0$ constant, the true terminal and young values at every public state equal those of the reoptimized normalized public reference with mean payments $(2, 2)$ and covenant (1). Actual normalized global optimizers independent of the macro history exist. All owner and GP histories, including owner-null histories and recalled private holdings, retain their full conditional comparisons.*

Proof. The household multiplier prices a date- j normalized payment X at physical value $A_t \beta^j \mathbb{E}_t X$. Its normalized personal utility is $\delta^j \mathbb{E}_t X$. Dividing each current payment, reserve and chronological bill by the known A_t therefore leaves the actual owner assessment, matched payoff, induced queue and discretionary budget unchanged. The treasury's genuine cost $\mathbb{E}_q C + B - W$ and the dealer's actual profit are multiplied by A_t . This does not evaluate a proposal at an unrealized target queue.

A price identity alone does not compare all stochastic contracts. Fix a current state and an arbitrary permitted stochastic menu. Carry its future macro path as an extra exogenous coordinate of its complete counterfactual service and financial schedules. Conditional on the current state, its law is independent of every physical effort/viability experiment. It can be nonatomic and correlated across dates. At each actual finite owner history and each remembered private holding, retain the joint law of the schedules for *every* compared effort plan. The allowed arbitrary post-effort mark law reproduces that joint pushforward. Each coordinate is revealed at the date when its original information or payment became known. A second-date aggregate state is not revealed at the first date. Schedules for different private actions are not replaced by independently resampled marginal lotteries.

For a terminal GP, all remaining consumption utility is linear. One may equivalently integrate the later macro coordinates in each complete conditional schedule. The service order intervals and joint aggregate/technical cap are convex, homogeneous and state-independent. They remain valid at $n = 0$ as well as $n > 0$. This operation preserves every conditional effort comparison, current owner-deviation payoff, actual source, and selected dealer-liability moment. Equation (3) gives the same gross productive source $(2, 2)$ under every physical effort law. No pure-high quote replaces the actual private mixture's deliveries. Ignoring the macro label except conversion to goods gives the reverse feasible inclusion. Hence terminal value is the same true $V(W)$ at every state, wealth and off-path book.

For a young GP it would be incorrect to average first-date wealth before applying its nonlinear continuation. Instead preserve its joint first-date kernel of first service, purchased current receipts, actual liquidation value $\beta \mathbb{E}_1 Y_2$ of assignable old later claims, and conditional unassignable late-service consumption. The latter remains a separately dated additive payment. The original issuer still owes the original $A_2 Y_2$ after the claim is sold. Its source is preserved by the actual tower

$$\beta \mathbb{E}_0 [\beta \mathbb{E}_1 Y_2] = \beta^2 \mathbb{E}_0 Y_2.$$

Further conditional mark coordinates reproduce late schedules only at their actual revelation dates. Full collateral and joint holding constraints survive before any law is compressed.

At each realized first-date wealth and public state, the already solved terminal problem bounds every continuation by $V(W)$ and a genuine reference optimizer attains it. Thus a private

old mark is not pretended to become a public signal to a new owner. That future public game is separately optimized at every state; its new physical clue is independent of old records. The resulting young comparison preserves the whole first-date wealth law and every actual owner, quote, private-deviation and source moment. It gives the reference upper bound, while an actual reference maximizer supplies the reverse inequality.

These operations precede finite-law reduction. A current macro state merely indexes one allowed finite current owner game; no continuum of new owner stages is inserted. Since the normalized problem is identical at every state, the same attained reference policy and its all-wealth Borel completion can be selected everywhere. No continuous optimizer is assumed. $\square$

The arbitrary mark domain matters. A prescribed smaller finite lottery alphabet need not contain the exact pushforward above. Finite support here is a derived optimal representation, after the full stochastic comparison; it does not exclude nonatomic financial deviations by assumption.

The optional publicly verifiable operation also remains available:

$$x \in [0, 1], \quad y(x) = \frac{7}{4} + \frac{x}{4}, \quad K(x) = \frac{333}{200} + \frac{x^2}{200}, \quad z_j \leq [y(x) - 1/4]n.$$

Outputs become $A_t[y(x) \mp d_t]n$. The physical posterior floor gives actual success probability at least $2/35$, while (3) gives mean pricing coefficient $36/25$. Replacing $x < 1$ by one keeps owner rewards, service and private books unchanged, and adds actual LP source at least

$$n(1-x) \left[\frac{(2/35)(36/25)}{4} - \frac{1+x}{200} \right] \geq \frac{37}{3500}n(1-x) > 0. \quad (4)$$

The extra input is paid from the extra LP receipts and any surplus stays with that fund. Applying this first to terminal optimization, and then to young optimization with that true terminal value, selects operation one without asserting strict GP preference.

1.2 A complete common economy with fixed populations

Write

$$I = \left[\frac{117}{640000} - 10^{-8}, \frac{117}{640000} + 10^{-8} \right], \quad \bar{Q} = \frac{205600000}{73121}, \quad \mu = \mathbb{E}_\pi A, \quad \eta = \mathbb{E}_\pi [Ad]. \quad (5)$$

All rates below use actual mean return dollars divided by mean beginning positions, rather than unweighted mean percentage returns.

Theorem 1.3 (A general-state public nonlinear common equilibrium). *Suppose Assumption 1.1 holds and $\eta > 0$. Independently choose primitives*

$$0 < \varepsilon \leq 10^{-6}, \quad F > \varepsilon \bar{Q}, \quad k \in I, \quad 0 < D < \frac{18\varepsilon}{55895} \frac{\eta}{\mu}. \quad (6)$$

Use the same actual founder mass F , GP birth mass ε , founder creation cost k and GP listing cost h in both countries. Set $B_U = 4 + D/2$, $B_E = 4 - D/2$ and $R_U = R_E = 1/50$.

There is a common stationary equilibrium with gross founder application value $u = k$, optimal mixed founder entry, strictly positive zero-cash young GP entry, voluntary endogenous information, optimal operation, nonlinear careers and finance by the same GP after actual high-effort success. The actual dated production and claim law generate its household SDF and optimal half-of-each-fund holdings. Country U is a statewise debtor with negative mean trade balance and a positive weighted return advantage.

If next-period d is conditionally nonconstant at every actual public history, those two fund holdings are unique there. This is payoff injectivity, not a claim that two securities span a general aggregate state space.

Proof. At the primitive $u = k$, solve the actual normalized public terminal and young programs with covenant (1). The strict active neighborhood in Corollary E.4 gives young net value greater than $3/80$ throughout I . Its program retains every unweighted conditional book comparison, voluntary owner history, actual private mixture, technical forecast and individual financier participation account. It has an attained optimizer and an all-wealth Borel senior completion. Lemma 1.2 proves that these are also the full stochastic values; an aggregate-independent normalized optimizer can be chosen.

Choose one such complete optimizing career. Let ν_k be its actual young successor law, with its actual private types, full history records, financial holdings, low/high effort and physical branch probabilities. Attach a genuinely optimal senior action at each successor. Nonfinance and unmatched paths remain present; old late service is distinct from liquid wealth. Its applications per original birth are

$$Q_* = q_y + \int q_s d\nu_k, \quad \frac{9}{3194} < Q_* < \bar{Q}.$$

For completeness, gross young utility satisfies $U \leq 1597m(q_y)/60$ and $m(q) < q/2$, so net value above $3/80$ implies $q_y > 9/3194$. With $a_0 = 317/500$ and $g = 197/500$, the same actual source bounds give $q_y < ca_0/k$ and $q_s < cg/k$. There is at most one senior posting per birth. Thus $Q_* < c(a_0 + g)/\min I = \bar{Q}$. These bounds apply to every selection. Each founder enters with probability $\lambda_F = \varepsilon Q_*/F \in (0, 1)$. At gross payoff k , entry and refusal both give zero net payoff. Entry precedes the independent physical clue, so its mass changes without selecting venture quality. Its actual mass is exactly εQ_* , and all used offers induce their actual queue at $u = k$. The slack creation-cost atom gives the positive-venue routing foundation with incumbent offers and commitments held fixed. No equilibrium for reoptimized finite-agent or coalition deviations is inferred.

Chronologically fresh independent population coordinates generate the actual two posting ages and consumption/payment ages through three. Each original venture keeps its success mark at both productive dates. Every claim retains its holder, issuer, original record and maturity as these change through payment or sale. The invariant history is the restriction of a bi-infinite flow of zero-wealth births. The selected normalized law is independent of macro innovations, so it combines with the primitive stationary Markov law. Nominal accounts use their actual payment-date A . No wealth reset or successor project supply is inserted.

The actual copied-book source inequality remains

$$\beta \int (V(W) - W) d\nu_{HS} \geq T + \frac{3}{500}N_H + \frac{259}{500}N_L > 0. \quad (7)$$

It forces finance after actual high-success by the *same* GP. A future option solely on no-current-finance paths would not establish this. The quantitative evidence and scale bounds give

$$N_y > \frac{9}{6388}, \quad f_{\min} := \frac{9\varepsilon}{55895} < f \leq 4\varepsilon. \quad (8)$$

Here f is actual successful-vintage scale per country/date, including both original productive dates and both posting ages. More than $719/2304$ of young financed scale uses an actual GP posterior above $3/5$. These statements follow from actual information-cell sources and hold for every optimizing career mixture. Initial refusal from positive h and the source bound, together with the positive option in (7), makes the true career value nonlinear.

The explicit path reserve is $R_{\text{path}} = 20008$. At each date four disjoint original-birth strips per country fund every permitted current path and old liability. The maximum age-one normalized call under the cap is $20000 + 4 + 7/4 + K = 20007.42$. The other payment ages have smaller bounds. Payments and the basic strip goods both scale by that date's A . Thus

$$B_c > R_c + 4R_{\text{path}}\varepsilon \quad (9)$$

backs gross conditional calls, not just expected bills. A sold claim retains its original issuer reserve; unused collateral stays in the same fund. Since $\eta/\mu \leq d_{\max} \leq 1/4$, the primitive gap in (6) is less than $9\varepsilon/111790$. Both basic outputs remain near four and (9) holds with strict slack.

Use the same selected normalized policy and the same actual population pair in the two countries. Equal population ratios alone would not suffice. Their actual investment, owner consumption and GP consumption, denoted i, o, g , are equal. Define

$$c_U = B_U + 2f - i - o - g - R_U, \quad c_E = B_E + 2f - i - o - g - R_E, \quad \bar{H} = c_U + c_E.$$

The national funds own the existing basic rights and their actual primary, treasury and dealer books once. Their dividends are

$$H_{U,t} = A_t(c_U - fd_t), \quad H_{E,t} = A_t(c_E + fd_t), \quad c_U - c_E = D. \quad (10)$$

The dated basic reserve gives positive national residuals. Financial transfers cancel only on their named accounts. Nonpecuniary costs and unused collateral are not counted again as consumed goods.

Let $\ell_\beta = \beta/(1 - \beta)$ and $v_c = \ell_\beta c_c$. The conditional mean-zero tower and the no-bubble transversality condition price the actual funds at

$$V_{c,t} = A_t v_c. \quad (11)$$

Initialize each household with half of each actual fund *after* settled date-zero consumption/distributions, with utility from date one. Half holdings consume $A_t \bar{H}/2$, satisfy Euler equations, and have discounted normalized wealth tending to zero. Their own SDF is $\beta A_t/A_{t+1}$, verifying the price used above. Bounded positive A gives finite ordinary log utility, and concavity plus the sequential budget proves global optimality.

For uniqueness, the two next total-payoff functions divided by positive A_{t+1} are

$$\frac{c_U}{1 - \beta} - fd_{t+1}, \quad \frac{c_E}{1 - \beta} + fd_{t+1}.$$

Their coefficient determinant in $(1, d)$ is $f(c_U + c_E)/(1 - \beta) > 0$. The two-column payoff map is therefore injective exactly when next d is conditionally nonconstant. Strict log and identical convex household opportunities give unique consumption even without spanning. For another optimum with the same initial wealth, the deflated difference of post-consumption wealth is a martingale. Nonnegative wealth and the candidate's bounded normalized wealth give

$$\mathbb{E}|M_t(W_t - W_t^*)| \leq 2\mathbb{E}[M_t W_t^*] \longrightarrow 0.$$

Conditional Jensen then makes wealth identical at every earlier actual history. Payoff injectivity forces the same quantities. When conditional nonconstancy fails, half holdings remain optimal but this quantity-uniqueness argument does not apply.

Finally, actual international accounts at half holdings give

$$\begin{aligned} NFA_t &= -\frac{\ell_\beta}{2} A_t D, & \mathbb{E}NFA &= -\frac{\ell_\beta}{2} \mu D, \\ TB_t &= A_t(D/2 - fd_t), & \mathbb{E}TB &= \frac{\mu D}{2} - f\eta, \\ \mathbb{E}RD_{\text{net}} &= f\eta - \frac{\mu D}{2}, & \mathbb{E}\Delta NFA &= 0. \end{aligned} \quad (12)$$

For each fund, return dollars are its actual next dividend plus next value minus beginning value. Capital gain is retained in every dated identity; only its stationary mean is zero. Thus $\Delta NFA = TB + RD_{\text{net}}$ statewise. The weighted rates are

$$\bar{r}_A = \frac{1 - \beta}{\beta} + \frac{f\eta}{\mu v_E}, \quad \bar{r}_L = \frac{1 - \beta}{\beta} - \frac{f\eta}{\mu v_U} > 0. \quad (13)$$

Both gross foreign legs are positive. The liability rate is positive because its actual national dividend is positive. Equation (8) and the independently fixed gap imply $D < 2f\eta/\mu$. The statewise debtor, expected deficit and positive weighted advantage follow. $\square$

Within this half-fund family the exact joint sign conditions are

$$D > 0, \quad \eta > 0, \quad D < 2f\eta/\mu. \quad (14)$$

Thus active repeated finance alone does not determine these signs. In particular, independent iid A and nondegenerate mean-zero d give active rank two but $\eta = 0$: a positive gap gives a debtor's mean trade surplus and equal weighted rates.

1.3 The complete same-institution comparison

Theorem 1.4 (Removing evidence under the general macro process). *Remove tests and every new viability signal, keeping the primitives, operating field, household securities and actual financial institution of Theorem 1.3. Allow all test-free communication and funding histories and aggregate-history-dependent candidates. The unique stationary real allocation in the stated bounded class is inactive. It is a full equilibrium with physically funded off-menu continuations, its own household prices, the same statewise NFA and positive mean trade balance.*

Proof. The actual finite-path reserves give every candidate normalized world household resources between

$$H_- = 8 - \frac{1}{25} - 8R_{\text{path}}\varepsilon, \quad H_+ = 8 - \frac{1}{25} + 8\varepsilon(2 + d_{\max}).$$

Consequently $H_- \geq 7.799936$, $H_+ \leq 7.960018$ and

$$\frac{H_+}{H_-} \leq \frac{3980009}{3899968} < \frac{6}{5}. \quad (15)$$

Identical initial half-world wealth and identical convex household opportunities imply identical consumption by strict log, without a spanning assumption. Their own normalized date- j price is $\beta^j H_t/H_{t+j}$, between $(5/6)\beta^j$ and $(6/5)\beta^j$.

The remaining actual GP posterior is at most $3/5$. Apply the positive price ceiling to actual output before taking its conditional mean. The mean-zero tower and independence from physical decisions give mean productive payment two at each original date under every selected scale and effort. With $F_H = 288/125$ and $F_L = 144/125$, actual high and low source deficits are

$$K - \frac{6}{5}F_H = \frac{139}{12500} > 0, \quad K - b - \frac{6}{5}F_L = \frac{8007}{12500} > 0. \quad (16)$$

For every $j \geq 1$, $(5/6)\beta^j > \delta^j$. Add the true selected GP, treasury and dealer accounts, retaining actual effort-dependent deliveries. Every terminal financed scale is strictly worse than consuming actual wealth. This includes mixed effort, private insurance and cross-history subsidies; no hidden physical profile is treated as an unavailable IC node. Writing N_H, N_L for the true expected high/low financed scales, the consolidated inequality is explicitly

$$U_s(W) - W \leq -\frac{139}{12500}N_H - \frac{8007}{12500}N_L - h\mathbf{1}_{\{\text{post}\}}.$$

Here U_s includes the low-effort private benefit; unused cash has unit value. Consumption at a future date costs at least its personal utility under the lower price bound. Nonproject financial positions obey the same inequality through actual counterparty participation. Refusal attains W , so terminal value is exactly W . For an assignable old later claim let P_1 be its actual normalized first-date liquidation value and P_0 its current value. The pricing tower and $m_{01} \geq \delta$ imply

$$\delta \mathbb{E}_0 P_1 \leq \mathbb{E}_0 [m_{01} P_1] = P_0.$$

The original issuer remains liable at maturity. An unassignable late service uses $\delta^2 \leq m_{02}$ directly and cannot fund a current premium. These comparisons extend the same domination to young GPs. Zero-wealth newborns refuse. Finite biological and original payment lives then eliminate every productive vintage from an invariant law.

The operating field is retained. Its high/low source-deficit derivatives are $x/100 - 648/3125$ and $x/100 - 324/3125$. They are negative on $[0, 1]$, so operation one remains the worst deficit and (16) covers every permitted operation.

For a positive counterfactual conserved venue mass v , actual founder entry solves the positive-cost cutoff correspondence

$$vq \in F \left[G\left(\frac{cR_F}{q+\ell}\right), G\left(\frac{cR_F}{q+\ell}\right) \right].$$

The homogeneous primitive $k > 0$ has the required cost floor. The unique continuous queue and full balanced-treasury conditional GP/forecast assessment in Lemma G.1 are unchanged by normalized macro conversion. At that queue every chronological call is physically reserved before individual financier participation is evaluated. A losing financier rejects the negative-source proposal before listing; its counterfactual acceptance has a real funded continuation. This does not impose active outside value k in the inactive market. Actual inactive founders face zero and refuse positive creation cost.

Only basic production remains. Its actual national dividends and prices are

$$H_{c,t}^0 = A_t(B_c - R_c), \quad V_{c,t}^0 = A_t \ell_\beta (B_c - R_c).$$

Half holdings supply half world consumption and satisfy their own Euler equations and TVC. The same primitive D gives the same NFA in (12), while mean trade balance is $\mu D/2 > 0$. Those securities are redundant. The selected constant half profile has equal weighted rates $(1 - \beta)/\beta$; that equality is not a conclusion for every optimal inactive portfolio. $\square$

A The owner game, balanced treasury and attained careers

The following bounded treasury institution is used on both sides of the comparison. Its actual owner assessment determines the queue of every proposed menu.

A.1 Primitive timing and actual current clearing

Keep the real two-date output technology

$$(p, p_0, \beta, \delta, b, K) = \left(\frac{4}{5}, \frac{2}{5}, \frac{4}{5}, \frac{1}{2}, \frac{1}{5}, \frac{167}{100} \right), \quad (y_1, y_2) = (2, 2),$$

with scale $n \in [0, 1]$, two GP posting ages and activation cost $h = 10^{-5}$. Use $m(q) = cq/(q + \ell)$, $c = 1/2$, $\ell = 1$, and $u = 117/640000$. A physical venue unit is available at each potential posting age; unused units remain idle, and their actual basic producer pays maintenance.

Applications occur *before* the private clue. After a match the owners share the equiprobable viable posteriors $2/5, 3/5$, send separate binary messages, choose founder then incumbent tests, receive both results only after both choices, voluntarily publish, report separately to the GP/dealer, and separately consent. The two good-only tests have sensitivity $1/2$ and costs $1/1000, 1/2000$. Owners retire with nonnegative fixed treasury-to-owner current receipt schedules, no owner-to-owner or owner-to-GP liabilities, no inherited wealth, no pledge of unvested fees and no new side contracts. The GP and its committed dealer receive the same signals; owners cannot observe private GP quantities or its seed. There is no intervening observable GP signal. No informed routing is inserted into the founder population. This receipt subclass is explicit, not a WLOG netting claim. Test, founder creation and GP activation costs are nonpecuniary. Test cost is incurred at the choice stage; its promised cash compensation is an actual owner receipt consumed at closing, unpledgeable beforehand. It is not an unfunded monetary test purchase or a second resource deduction. Venture input and basic maintenance are real goods costs. Owner cash settles after technical effort, privately to the treasury and recipient. Its payment record is not an additional GP/dealer signal beyond the specified publications and reports; altered reimbursements do not silently refine the technical information partition.

Retain the bounded current-cash institution of section D, the same aggregate purchased-payment cap four with technical component at most $4n$, and service at most $2n$ at each productive date. Genuine nonproject claims use the same cap and cannot retain a nonexistent technical failure trigger. Input/service obligations are irrevocably funded before optional technical orders. The GP may freely rebalance actual prepaid current claims; its later service is unassignable. The independent post-effort payment mark and the pre-test public activation coin retain their distinct dates.

Lemma A.1 (Balanced funding at the actual owner-induced queue). *A cash proposal fixes its owner, input, service and mandatory dealer rules, and a discretionary current GP budget $B \in [0, 50]$. At wealth $W \in [0, 6]$ the actual prior issuer credits and assignable-claim sale proceeds are first settled and their total recorded to the treasury and dealer. A newborn's recorded wealth is zero. Pre-offer consumption, transfers, observable burning or unrecorded outside cash are excluded at this stage. This is an explicit timing restriction, not a WLOG claim for hidden-wealth or earlier-consumption domains. All private consumption and trading begin afterwards from B . Let $a = (W - B)_+$ and $f = (B - W)_+$. Conditional on funding, the GP pays a to the primary treasury or receives f , leaving exactly B liquid. Stage one reserves these GP-treasury directions for the canonical a/f rows; optional private current-event orders begin at stage two. The treasury separately backs every actual mandatory branch call C_h . Its individual expected current net cost is*

$$L(q) = \mathbb{E}_q C_h + B - W. \quad (17)$$

If R_F is the selected cash-owner assessment's expected matched founder utility after test cost, the actual positive queue is

$$q = \max\{cR_F/u - \ell, 0\}. \quad (18)$$

Every induced positive queue has a physically funded follower assessment; actual individual LP/dealer participation determines acceptance. Canonical $q = 0$ is no listing, with no activation or treasury call.

Proof. The owner's conditional matched game is independent of q and technical quotes because its cash payments are fixed. Select a consistent owner assessment first, and use its actual R_F . If $q > 0$, $[c/(q + \ell)]R_F = u$. Use that queue in every matching and current-event price. The treasury pays *actual* C_h from its named branch reserves, and $f - a = B - W$ fixes the GP's real discretionary source independently of q . Zero optional purchases remain feasible on each history. The number L is an expectation in the investor's participation row, not unbacked cash presumed to cover a larger conditional call. A negative L is an actual receipt, not extra physical investment.

Apply theorem D.1 to this actual queue. Its complete-book forecast fixed point gives optimal financial/effort behavior at every owner history. The LP's profit is its actual retained-output value minus the full L . The dealer's profit uses actual mandatory receipts and private premiums minus effort-correlated liabilities. Optional premiums are not added to C_h as fictitious primary cash. If a completion has both profits nonnegative, select one and let both financiers simultaneously commit before listing. Otherwise select a funded completion and let a losing financier reject; its unilateral acceptance would trigger that physically possible losing program. No rejected package obtains a queue, input or technical claim.

If $R_F \leq u\ell/c$, use canonical no listing. Equivalently a funded zero-match package cannot improve refusal: individual participation gives $B \leq W$ after nonnegative uses, and without a technical project terminal fair claims cannot beat cash. A zero-wealth young agent receives no free financial source. Thus no impossible funded continuation is hidden in rejection. $\square$

For measurable completion, the active follower graph is considered on compact slices $q \geq 1/k$, $k = 1, 2, \dots$. Funded and jointly profitable slice projections are closed, so their unions are Borel. Choose the first feasible profitable slice, or otherwise the first funded slice and a rejecting financier, and apply the nested-ball selector within that compact fiber. If no owner assessment induces positive q , select from its closed zero-queue owner set and use canonical refusal. This does not assert global closedness of the call-erasing $q = 0$ normalization.

For maximizing policies the stronger source bounds keep active queues away from this boundary. A senior listing weakly beating refusal satisfies $m(q)g \geq h$, $g = 197/500$, and hence

$$\frac{1}{19699} \leq q_s < 400.$$

The public young source bounds current consumption plus household-priced personal claims and owner cash by $a_0m(q)$, $a_0 = 317/500$. Since $V(w) \leq 200w/3$, its gross utility is at most $m(q)[(125/3)a_0 + b] = 1597m(q)/60$. The strict benchmark exceeds $3/80$; therefore every relevant maximizing sequence satisfies

$$\frac{9}{3194} < q_y \leq \frac{ca_0}{u} - \ell = \frac{202763}{117}. \quad (19)$$

No certificate-only activity bound is used.

A.2 Attained careers and their own source law

Lemma A.2 (Actual continuation and a strict young source). *The balanced current-cash senior value V_C is attained at every $W \in [0, 6]$, has a Borel complete policy, and is nondecreasing*

and Lipschitz 200/3. The full young accepted menu problem, including every conditional financial/effort deviation and actual financier participation, attains a strictly positive maximum. Its actual successor law has finite support.

Proof. Complete-book conditional optimality and the conditional financial-continuity lemma give closed active assessed graphs. The bounds above, finite cash and collateral template, and continuous balances (17)–(18) give compactness of all weakly improving senior and relevant young active programs. Refusal is a separate action. Thus no discontinuous call switch at $q = 0$ is used to prove attainment.

The source proof of rent-restoring thinning survives the new treasury rule. For an old program with source W , discretionary budget B and owner cost uq , activate its conditional history with probability λ and pay the same founder rent when inactive. Set

$$W' = \lambda W + (1 - \lambda)uq, \quad B' = \lambda B, \quad \mathbb{E}C' = \lambda \mathbb{E}C + (1 - \lambda)uq.$$

Then $L' = \mathbb{E}C' + B' - W' = \lambda L$. Conditional active payments and quoted prices stay unchanged; their actual probabilities and premiums scale. An old conditional current claim retains its actual bound50 and issuer reserve, not a purported bound by initial wealth six. Dealer accounts scale in the same way. Founder value and hence q stay unchanged. The full-history reverse-map argument yields Lipschitz constant 200/3. Increasing W while keeping B and the real continuation fixed reduces L , preserving financier participation and proving monotonicity. The canonical rows remain within $a \leq W \leq 6$, $f \leq B \leq 50$, so this does not enlarge an already saturated access-payment row. Nested closed-ball selection gives a Borel optimizer; choose refusal at zero gain and complete all other menus by theorem A.1.

The exhausted-source offer retains

$$W_* = \frac{3363}{640000}, \quad q_s = 19, \quad t_s = \frac{57}{320}, \quad V_C(W_*) \geq \frac{1781}{25000}.$$

With $B = 0$ its treasury cost is $t_s K + T_s - W_* = t_s(K - 3/500)$, exactly the actual retained LP value. The public source upper is unchanged by treasury transfers, which cancel in the actual accounts. The zero-wealth young offer has $q_y = 1$, $t_y = 3/32$, $T_y = 357/640000$, $B = A = z_2 = 0$, unit certified scale, and first-date success service 2 with probability $624687/1276637$ and W_* otherwise. Its mean is $12561/12800$. The actual treasury cost $t_y K + T_y$ equals retained LP value because $T_y + t_y p \beta \mathbb{E}Z_1 = t_y(F_H - K)$. Its worst matched input/owner call is $267557/160000$. Using the attained $V_C(W_*)$ and $V_C(2) \geq 2$, each actual certificate cell has IC slack $9590959/3191592500 > 0$, and net entry utility is at least $9716088611/255327400000 > 3/80$. No free current premium can be financed from its unvested service.

Use this actual V_C in the complete young program. The two-stage theorem theorem D.3 preserves every recalled source, conditional utility, alternative-plan dual, owner deviation and actual quote/dealer moment. Its uniform finite type/late-mark bound and (19) prove attainment anew. The benchmark is not asserted to be the optimum. The value is nonlinear: $V_C(0) = 0$, the source bound and h imply refusal on $[0, 3h/197]$, whereas $V_C(W_*) > W_*$. $\square$

Positive young entry also proves actual information use. Adding its primary, dealer and current source rows at zero initial wealth gives

$$C_0 + \text{PV}(\text{dated personal consumption}) + T_y \leq \sum_h \omega_h n_h (\alpha_h F_{\text{mix},h} - K), \quad F_{\text{mix},h} \leq F_H = \frac{288}{125},$$

where h is an actual GP information cell. Since $T_y \geq uq_y > 0$, a positive-mass financed cell has $\alpha_h > 835/1152$. Without acquired and used evidence every available physical profile, and hence every actual GP posterior, is at most $3/5$; but $(3/5)F_H = 864/625 < K$. Thus optimizing young contracts acquire and transmit useful evidence on positive mass. This does not impose IC at a

hidden profile or use publication counts as a proxy for informativeness; informative suppression and reports are included.

Choose one actual young optimum. If a_r are its private complete-type weights, w_j its independent post-effort mark weights and $P_{r,e}$ the *true* physical history law under type r , let $Y_{r,ej}$ include first-date purchased receipts and the contemporaneous market value of all old assignable later claims, each with its original issuer retained. Its successor law is

$$\nu = \sum_{r,j,e} a_r w_j P_{r,e} \delta_{(Z_{1,ej} + Y_{r,ej}, Z_{2,ej}, \text{actual records})}. \quad (20)$$

It has mass one and wealth in $[0, 6]$. There is no P/Q factor because physical probabilities remain outside the independent type and mark laws. Unmatched, nonfinanced, low and failure histories remain. Old unassignable Z_2 adds its separately dated utility at age one and cannot affect senior cash or posting. A sold old financial claim retains its original issuer and payment.

Let ν_{HS} be the actual high-success submeasure. The full-history source and $J(w) = V_C(w) - w \leq g\mathbb{E}[N_s | w]$ give, for actual senior financed scale,

$$\beta \int J d\nu_{HS} \geq T_y + \kappa N_H + (K - F_L)N_L \geq T_y + \kappa N_y, \quad (21)$$

$$R := \int \mathbb{E}[N_s | w] \nu_{HS}(dw) \geq \frac{T_y + \kappa N_y}{\beta g} > \frac{15}{788} N_y > 0. \quad (22)$$

Here $T_y \geq uq_y > 0$. This quantitative successful same-GP renewal holds for arbitrary actual Borel wealth laws and measurable kernels, not just finite support. It does not count wealth generated only after nonfinance as a successful lineage.

B Terminal source bounds and a funded benchmark

B.1 A voluntary exhausted-source optimum

Set $\Delta = p - p_0$, $B_g = pb/\Delta = 2/5$, $C_1 = 16/25$, $\kappa = 3/500$, and $g = 197/500$. Copying an actually selected high financial policy gives discounted *dated* personal consumption $D_H \geq B_g n$; current cash cancels in that comparison. Its household-priced cost is at least $(\beta/\delta)D_H$. Summing actual LP and dealer accounts over the selected high and low policies yields

$$\begin{aligned} W &\geq \kappa N_H + (K - F_L)N_L + \text{other nonnegative uses,} \\ U - W &\leq gN_H - \frac{159}{500}N_L - \text{owner cash,} \quad F_L = \frac{144}{125}. \end{aligned} \quad (23)$$

Here N_H, N_L are actual expected financed scales, including effort mixing. Unknown viability can only reduce the output used in these bounds. Hence $V_{\text{pub}}(0) = 0$ and $V_{\text{pub}}(W) \leq (200/3)W$. This inequality alone is not a Lipschitz theorem.

The finer source argument first imposes IC at actual GP cells and then expands the complete private profile. Its high coefficient is g at an actual positive and negative without one; its low coefficient is always negative. Both delayed tests uniquely maximize the resulting upper source, giving

$$S_* = \frac{3}{8}g - c_F - c_I = \frac{117}{800}, \quad U - W \leq m(q)S_* - uq.$$

This bound permits private reporting, voluntary withholding, different public effort cells and funded subsidies between histories.

Proposition B.1 (An attained voluntary-publication peak). *At $W_* = 3363/640000$ the true complete-history value is*

$$V_{\text{pub}}(W_*) = \frac{57}{800} - h = \frac{1781}{25000}.$$

It is implemented with $q_ = 19$, both delayed tests, voluntarily published positives, pooling independent messages/reports and selected consent.*

Proof. The directed source maximizer has

$$m_* = \frac{19}{40}, \quad r_* = \frac{117}{16000}, \quad t_* = \frac{57}{320}, \quad T_* = c_F + c_I + r_* = \frac{141}{16000}.$$

An actual assignment claim delivers $T_* + (3/8)\kappa$ if matched, at price $m_*[T_* + (3/8)\kappa] = W_*$. Its issuer reserves the full liability. The menu pays the founder $c_F + r_*$ and the incumbent c_I after both performed tests. These earned payments survive publication, reports, consent or veto and every GP portfolio choice; they are zero if a test was skipped. They are funded before testing and settle at current closing.

The founder strictly tests. After founder testing the incumbent is indifferent and testing is selected; after a skip it strictly skips. Messages and reports pool and are ignored. Publication and consent are weak best replies because they leave each earned receipt unchanged. These comparisons hold after every hidden GP financial deviation, whose quantities do not affect the owners' contracts. They do not establish strict or unique publication.

After the recorded test choices, the remaining $(3/8)\kappa$ buys a claim delivering κ on an actually published certificate. It is priced under the selected voluntary-publication strategy, with the real conditional liability reserved. No GP action can fabricate its trigger. At a certificate and selected consents, the GP contributes κ , the LP $K - \kappa = 208/125$, and service is $(z_1, z_2) = (1, 0)$. Actual residual output $(1, 2)$ has value $p(\beta + 2\beta^2) = 208/125$, so LP participation binds. Input is deposited before hidden effort and technical insurance.

Each financed unit gives B_g under either effort. After the owner obligations, available source has value $t_*\kappa$. Every optional technical financial dollar has utility/price at most $15/8$, even under a selective effort plan over the separate certificate records. Earlier fair current trades telescope with their actual effort-invariant laws. If F_{fin} denotes priced *nonproductive residual uses*, excluding intermediate claims already counted as cofinance, then

$$\kappa N + F_{\text{fin}} \leq t_*\kappa, \quad U \leq B_g N + \frac{15}{8} F_{\text{fin}} \leq t_* B_g,$$

since $15/8 < B_g/\kappa = 200/3$. Funding every certificate and buying no optional technical hedge attains the upper bound. The separate global menu source bound is also attained, proving optimality at this wealth.

For consistency, take pooling message/report trembles independent of the clue and test results, and clue-independent trembles around the prescribed test choices. A published authentic positive sets viability to one; joint silence after both tests has posterior $1/5$. The finite common-tremble Bayesian limit completes every remaining information set. Unused reserves on a skip or veto return to their actual funders, and released current GP cash is consumed when its committed menu has no productive branch. A veto never produces a technical failure claim. Owners' cash comparisons are independent of these off-path beliefs. $\square$

B.2 A funded public-activation regularity theorem

Add an immutable contractible activation coin after matching and *before* testing, with its current insurance arranged before the draw. When inactive, the founder receives its promised matched net rent $r(q)$, the incumbent receives zero, and no productive action is instructed. Optional tests waste their cost and cannot change the fixed receipt. When active, play the original full menu and assessment. This is an explicit new timing institution, not a reinterpretation of the later compensation lottery. It is closed under further thinning: multiply activation probabilities and merge the identical inactive branches; do not merge the base menu's endogenous reporting histories.

Theorem B.2 (Public-activation continuity from actual sources). *On any wealth domain whose current activation cash liabilities are actually backed, the optimized value V^T in this enlarged*

institution is nondecreasing and globally 200/3-Lipschitz. This holds for its supremum value without assuming terminal attainment or a selector. Refusal is feasible at every wealth, and the funded benchmark at W_ remains available.*

Proof. Take an implemented profitable senior policy at W , queue q , and gross GP payoff U before h . Put $a = uq = m(q)r(q)$. For $0 < \lambda \leq 1$ its thinned current source is

$$W' = \lambda W + (1 - \lambda)a.$$

Every original matched active history retains its conditional quantities and has probability multiplied by λ . Matching itself is unchanged: original unmatched current GP uses are instead scaled in quantity. The added inactive founder payment has expected source cost $(1 - \lambda)a$. Founder expected net matched rent is still $\lambda r + (1 - \lambda)r = r$, so the queue remains q . Every active owner incentive is unchanged and the inactive rent has a real prior reserve.

Actual current activation claims bought before the coin deliver the old conditional active holdings; their issuers reserve the full liability. Post-coin conditional prices stay unchanged. It is their unconditional premium, not their conditional quote, that scales by λ . After binding owner/input/access accounts, common free financial value is λB . The entire contingent technical book, actual correlated dealer loss and each selective effort plan therefore scale by λ in probability, with the pointwise cap still four. In the reverse mapping, inactive current consumption becomes original set-aside current cash after division by λ . Inactive dated financial payments can be replaced by current cash at their actual conditional β prices, weakly improving terminal utility because $\beta > \delta$. These transformations use only current or vested assignable sources, never unvested service. Active service stays on its original date. Thus the true gross optimal private payoff of this fixed thinned assessment is λU , including all contingent financial deviations.

The actual source proof strengthens (23) to

$$W - a \geq \kappa N_H + (K - F_L)N_L, \quad U \leq W - a + gN_H - \frac{159}{500}N_L \leq L(W - a), \quad L = \frac{200}{3}.$$

For $W_1 < W_2$, consider any profitable policy at W_2 ; then $W_2 > a$. If $W_1 > a$, take $\lambda = (W_1 - a)/(W_2 - a)$ to obtain

$$(U - h) - V^T(W_1) \leq (1 - \lambda)U \leq L(W_2 - W_1).$$

If $W_1 \leq a$, the direct source bound gives $U - h \leq L(W_2 - a) \leq L(W_2 - W_1)$ and $V^T(W_1) \geq 0$. No zero-probability active assessment or division by zero is needed. Refusal has slope one. Taking the supremum proves the upper difference bound, even along a nonattaining sequence. Increasing wealth can instead be paid as an additional actual mandatory dealer amount, preserving disposable budgets, strategies and quotes while relaxing its participation. Hence V^T is nondecreasing, proving

$$0 \leq V^T(W_2) - V^T(W_1) \leq \frac{200}{3}(W_2 - W_1).$$

The source bounds remain valid with the activation coin: its inactive rent is a nonnegative use and creates no evidence or output. The benchmark uses activation one. $\square$

C Conditional financial continuity

C.1 An exact financial value continuous at the boundaries

Fix an outer menu and assessment. The finite exclusive effort cells have probabilities ω_i , common posteriors α_i , and committed success quotes

$$\pi_i = \alpha_i \left(\frac{2}{5} + \frac{2}{5}\lambda_i \right).$$

Here λ_i is the assessed actual high-effort frequency. Let a be any pure contingent effort plan, selecting high or low at every cell. Let B be the actual common market value left after fixed input, owner and mandatory dealer obligations.

All current trades telescope at their assessment prices. Conversely, an affordable complete future book can be prepaid, with the actual issuer delivering its reserved claims at each branch. Thus the current-event cone implements every prefix of that book from B . It does not finance a premium by future unpledgeable service. For a terminal GP, an effort-invariant dated nonproject claim is strictly dominated by current consumption, since $(\delta/\beta)^d < 1$. Removing it releases premium and relaxes the aggregate cap. It is therefore absent from every terminal optimal book, although it remains available to a young GP with nonlinear continuation.

For coordinate $l = (i, \xi, d)$, where $\xi \in \{S, F\}$ and $d \in \{1, 2\}$, define

$$c_l = \omega_i \beta^d \Pr(\xi), \quad v_{a,l} = \omega_i \delta^d \Pr_{\alpha_i, a_i}(\xi). \quad (24)$$

Let K_a denote expected service utility and private effort benefits. Fixed nontradeable background consumption is kept separately. The exact pure-plan financial value is

$$F_a = B + K_a + \max_{\substack{y \geq 0, \ c^\top y \leq B \\ y_{i,\xi,1} + y_{i,\xi,2} \leq 4n_i}} (v_a - c)^\top y. \quad (25)$$

Unspent premium is current consumption. Each contingent effort plan has one complete budget, rather than a separate budget for each public certificate cell.

Lemma C.1 (Boundary-continuous financial optimization). *On a compact outer parameter set, F_a is continuous in $B, n, \omega, \alpha, \pi, K_a$, including zero cash, zero prices and zero funded scale. Its bounded maximizing-book graph is closed. The same conclusion holds for the maximum over the finitely many contingent effort plans.*

Proof. For every feasible actual quote, the technical likelihood ratios under either compared effort satisfy

$$\frac{\Pr_{\alpha,a}(S)}{\pi} \leq 2, \quad \frac{\Pr_{\alpha,a}(F)}{1-\pi} \leq 3.$$

If $\pi = 0$, then $\alpha = 0$, so the actual success coefficient also vanishes. If $\omega_i = 0$, both unconditional coefficients vanish. Hence, including these boundaries,

$$0 \leq v_{a,l} \leq \frac{15}{8} c_l. \quad (26)$$

The analogous actual household-priced liability is at most $3c_l$.

The feasible set is nonempty, compact, and has closed graph. Maximizing subsequences prove upper semicontinuity. At a target with $B > 0$, take any feasible y . For each $n_i > 0$, multiply its cell coordinates by $\min\{1, n_i^k/n_i\}$; coordinates at $n_i = 0$ are already zero. The resulting $\tilde{y}^k \rightarrow y$ satisfies the nearby caps. Multiply the whole book by

$$t_k = \min \left\{ 1, \frac{B_k}{c_k^\top \tilde{y}^k} \right\},$$

using $t_k = 1$ at a zero denominator. A necessary correction has $t_k \rightarrow 1$: limiting expenditure is either below B , or equals the positive B . The nearby feasible book converges to y , proving the lower value bound.

At $B = 0$, free-price quantities have zero actual utility by (26), and for all $B \geq 0$,

$$0 \leq \max_y (v_a - c)^\top y \leq \frac{7}{8} B. \quad (27)$$

Thus excess financial value tends to zero without dividing by a vanishing price. This proves continuity at zero cash even though the feasible quantity correspondence itself need not be lower hemicontinuous there. Continuity and closed feasibility imply a closed maximizer graph. A finite maximum preserves continuity. $\square$

The cap also settles the physical boundary. As $n_i \rightarrow 0$, its technical payments tend to zero. A dummy zero-payment technical label can then be erased; it supplies neither output nor a priced failure event. Any genuine nonproject claim keeps its actual nontechnical trigger. There is no further current decision after the terminal technical mark.

Lemma C.1 applies conditionally at any recorded GP decision history, using its actual conditional physical belief and the market value of its recalled assignable positions. Those are parameters; an unconditional probability of zero is not used to delete the conditional comparison.

C.2 Complete private strategies before owner incentives

An ex-ante maximizing book is sequentially optimal at its reached private histories: a profitable continuation could be spliced into a profitable complete-plan deviation. This argument does not cover an owner history of probability zero. Nor does averaging books within a fixed effort plan solve the problem. Although that plan's ex-ante optimal LP face is convex, the average recalled wealth can have a different conditional effort maximizer.

We therefore retain complete contingent books before imposing the owner constraints. Let x collect the bounded outer menu, owner behavior, conditional beliefs, quotes and scales. A private seed selects a book s containing intended financial controls and effort at every physical owner history, including null histories. The book retains the private holdings recalled along each such history and obeys all actual prefix and capacity constraints. Its effort labels lie in a finite discrete set and its amount coordinates in a compact box.

At a GP decision history I , let $B_I(x, s_{\text{past}})$ be the liquidation value of its actual recalled positions after fixed obligations. Let $D_I(x, s_{\text{past}})$ be its conditional optimum, computed from the conditional version of (25) and all contingent effort plans. The funded current-event cone supplies that reduction at every history. Conditional belief coordinates belong to the assessment itself; no division by the equilibrium probability of reaching I is required. Lemma C.1 makes D_I continuous. Let $U_I(x, s)$ be the prescribed book's actual conditional payoff.

Definition C.2 (The complete sequentially optimal book graph). The set $\Gamma(x)$ consists of feasible complete books satisfying

$$U_I(x, s) = D_I(x, s_{\text{past}}) \quad \text{at every GP decision history } I, \quad (28)$$

including histories reached by owner deviations and the initial financial choice. These equalities are not multiplied by the equilibrium probabilities of their histories.

The graph of Γ is compact: it imposes finitely many continuous equalities and inequalities on the compact amount and effort domain. It may be empty for an inconsistent x ; nonemptiness is not assumed for every outer assessment.

The GP's private seed initially is independent of physical viability and results. After posting, no observable GP action intervenes before the owner history, and owners cannot see private quantities. Consequently every owner-history likelihood is independent of this seed. Its conditional law remains its initial law, including at null owner histories in a consistent full-support tremble limit. The history likelihood cancels from the posterior ratio. This argument would not apply if a GP observable action could signal its private seed during the owner tree.

Retain a law μ supported on $\Gamma(x)$. At every owner history i , including null histories, impose the actual quote

$$\pi_i = \alpha_i \left[\frac{2}{5} + \frac{2}{5} \int \mathbf{1}_{\{a_i(s)=H\}} d\mu(s) \right]. \quad (29)$$

The same seed law enters every owner's conditional deviation payoff. Owner deviations change physical histories but do not make the GP privately depart from its prescribed book. Thus the law faced at all of their resulting histories has already been subject to (28).

Private recalled portfolios generated by an earlier GP deviation may lie outside the prescribed books. At those states a parametric Borel optimal completion is required. The conditional

finite LP bases provide it: choose the first feasible optimal basis on its semialgebraic region, with lexicographic selection on degenerate optimal faces. This rule is applied to every feasible recalled portfolio. It cannot change an owner's deviation payoff, because every prescribed seed and owner-history pair is already inside the complete book law. This is the limited, valid role of the additional completion.

D Complete follower assessments and finite-law attainment

The follower construction completes every admissible cash menu, including rejected menus and owner-null histories. The final treasury in Appendix A funds each chronological mandatory call at the queue induced by the actual owner payoff.

D.1 The bounded current-cash institution

Use the actual information and production primitives of section B. Fix the finite owner history and report alphabets. Each terminal owner's contractual receipt is a fixed current cash amount at its finite history, reserved before tests. It has no technical-outcome, GP-effort or posterior-liquidation-price component. Owners have no outside wealth, cannot pledge unvested fees and cannot write new side contracts. They do not observe the GP's private quantities or randomization seed. There is no intervening observable GP action before the owner history is complete. Every signal received by the GP is also received by its committed technical underwriter.

The physical primary scale, input, owner payments and mandatory access obligations are contracted and funded before optional technical orders. The two efforts remain physically available. Complete prepaid current-event claims span the effort-independent owner, matching and public activation events, with actual worst-path backing. They liquidate assignable positions at their committed current-event values; they neither pledge service nor create a GP-controlled failure. Conditional current source after fixed obligations is feasible independently of the technical quote vector.

Each current cash or commitment coordinate lies in $[0, 50]$. Within the *same* aggregate beneficial remaining-payment cap four, the technical component at scale n is capped by $4n$. A genuine nonproject component uses that same capacity and cannot depend on a nonexistent technical success/failure when $n = 0$. Service is at most $2n$ at each real productive date. Current activation assignments have their actual reserved rows. Thinning copies an old conditional current claim within its original bound50 and issuer reserve; initial wealth at most six does not bound a concentrated conditional delivery. The inactive founder receipt has its own bounded row. These are explicit collateral restrictions; an expected source inequality does not imply them. The fixed tree has a finite gross reserve R_{pub} , obtained by summing its actual bounded liability rows.

A physically admissible proposal satisfies these statewise source and reserve constraints, without imposing financier profitability. Zero optional purchases remain feasible on each history if its package is funded. The LP and dealer simultaneously commit before listing the offer for directed applications. A rejected proposal receives no queue; its proposed q describes only the funded continuation if accepted. No cash is called and no technical claim exists until both commit. Rejection returns unused reserves and ends the proposal. A contrary acceptance triggers the specified genuinely funded continuation.

For subsequent compact optimization the unrestricted-public young source bound, rather than a certificate-only probability bound, gives

$$uq \leq a_0 m(q), \quad a_0 = F_H - K = \frac{317}{500}, \quad q \leq \frac{ca_0}{u} - \ell = \frac{202763}{117}.$$

Thus its matched rent is at most a_0 . The sharper senior information-source bound gives $q < 400$. Both fit the stated cash caps.

Theorem D.1 (A follower equilibrium after every cash menu). *Every physically admissible cash menu has a complete follower assessment, including consistent owner beliefs, conditional optimal GP financial and effort behavior, technical quote consistency at every owner history, and actual financier acceptance or rejection. For a funded assessment with J GP effort histories, at most $J + 1$ complete private books suffice to witness technical forecast consistency. No continuous assessment selector is assumed. The result applies to every fixed admissible young service law, including an arbitrary independent post-effort law.*

Proof. First ignore financier profitability. The owners have a finite perfect-recall cash-payoff game independent of technical forecasts and GP portfolios. Put positive lower bounds on finite action probabilities, take agent-form Nash equilibria, and pass to a common limit of strategies and Bayesian beliefs. Conditional one-shot inequalities survive; perfect recall gives full sequential optimality. Fix the resulting consistent owner assessment. Extend its same tremble sequence to the GP/dealer information partition. Write α_j for their induced common viable posterior at every effort history, including null owner histories; an owner's unreported certificate is not inserted as GP information. Let

$$\mathcal{P} = \prod_{j=1}^J [\alpha_j p_0, \alpha_j p].$$

For each committed $\pi \in \mathcal{P}$, the GP has a single-agent finite history problem with genuinely funded primary obligations. Its conditional financial value is continuous at every recalled source, including zero budget and zero scale. In the young case this follows from continuity of the actual V on $[0, 6]$, the common-budget law formulation and its attained dual in $[1, 125]$. Conditional true/quote likelihood ratios are at most three; zero quote then also has zero true payoff. Trimming technical claims with n and rescaling the common optional budget establishes the necessary value continuity. A generic inaccessible-receipt prefix system would not justify this step.

Dynamic programming gives a complete private book optimal at every owner history and its own recalled holdings, including null histories, and jointly optimal initially. Let $\Gamma(\pi)$ contain these books. The domain consists of bounded current coordinates, finite effort-plan labels and, when needed, compact weak financial kernels with the fixed service marginal. It is compact. Unweighted conditional optimality equalities and continuous values make Γ have a closed graph; dynamic programming makes its values nonempty. Do not replace a law of these books by its mean holdings.

For $s \in \Gamma(\pi)$ define

$$t_j(s) = \alpha_j [p_0 + (p - p_0) \mathbf{1}\{s \text{ chooses H at } j\}], \quad \mathcal{F}(\pi) = \text{co}\{t(s) : s \in \Gamma(\pi)\}.$$

The private seed is independent of the owner-history likelihood. Its conditional law is therefore its initial law at every owner history, including consistent limits of null histories. The map $\mathcal{F}$ has nonempty compact convex values in $\mathcal{P}$. Carathéodory represents each value by at most $J + 1$ books. Taking subsequences of these books and weights proves its closed graph from that of Γ ; compactness gives upper hemicontinuity. Finite-dimensional Kakutani supplies $\pi^* \in \mathcal{F}(\pi^*)$. Its representing law consists of complete sequentially optimal books and reproduces every conditional forecast. The late service mark is not observed earlier in this construction.

Now compute both financiers' actual profits from that complete law, including effort-quantity correlation. If some funded assessment has both profits nonnegative, select it and let both financiers commit. Otherwise select any funded assessment, which exists by the preceding argument, and let a financier with strictly negative profit reject. Its unilateral acceptance would trigger that losing continuation, so rejection is optimal. Commitments are simultaneous, so the other financier pays nothing while the package is rejected and is indifferent between signing and declining. After rejection no test, input or technical claim is funded. This completes every menu without omitting unprofitable proposals from the game. $\square$

Lemma D.2 (Borel completion across menus). *On the fixed bounded template the follower assessments and acceptance rules in theorem D.1 can be selected measurably across menus.*

Proof. The complete-book optimality graph, finite owner consistency and all actual quote and participation moment constraints are closed in compact domains. Retain finitely many complete-book moments, including both financier profits; for a young law use the sequential reduction below. Thus the funded follower graph is compact. Adding the two nonnegative-profit inequalities gives another compact graph, whose projection onto menus is closed.

For either graph, recursively choose the first closed ball from a countable dense family, with radii decreasing to zero, that meets the fiber and all previously chosen balls. Each feasibility test is the projection of a compact set, hence is Borel on each previously selected index cell. The nested nonempty compact intersections have vanishing diameter and determine a Borel limit selector. Choose the profitable selector on its closed menu projection, and otherwise choose a funded selector and the first financier with negative profit. No continuity of this selection is asserted. $\square$

D.2 Arbitrary young laws with all conditional incentives retained

Theorem D.3 (Sequential two-stage payment reduction). *Fix the bounded history/account template. Suppose the true terminal value V is continuous on $[0, 6]$ with Lipschitz constant $200/3$. Every implemented young assessment has an equivalent assessment with finitely many private complete types and finitely many common post-effort mark atoms. It preserves all conditional GP incentives, owner deviations, forecasts, source and actual dealer accounts. In particular, if d_0 external moments suffice for private-type compression, at most $R = d_0 + 1$ private types are needed. A sufficient subsequent service-mark bound is*

$$N \leq 1 + d_{\text{ext}} + \sum_{r=1}^R \left[p_r + H + \sum_{h=1}^H K_h \right], \quad (30)$$

where p_r counts retained conditional source/prefix/liquidation moments, K_h is the number of conditional effort plans at GP history h , and d_{ext} includes every other required quote, owner-deviation and financial moment. The bounded accepted young value is attained.

Proof. Fix the common complete service law μ , whose mark is resolved only after effort. For each conditional plan, purchased payoffs form a law with marginal μ and support in the whole linked payoff polytope. This retains the shared nonproject component and technical cap $4n$ inside aggregate capacity four. The value is affine in that law, even though V is nonlinear. One common conditional premium budget has an attaining scalar dual $\lambda \in [1, 125]$ because $(\delta/\beta)(200/3) \max(P/Q) \leq 125$. The dual integrand maximizes over the complete linked schedule, rather than separately across outcomes.

For nearby service laws, couple their coordinates at vanishing mean distance, retain the nonproject component, trim technical components with the scale and restore the common premium budget by uniform shrinking. At positive limiting budget the last factor tends to one; at zero budget the financial excess is at most 124 times that budget. This proves joint conditional value continuity and a closed graph of complete sequentially optimal types. All recalled-source expectations are included; service is never treated as assignable cash.

First compress the law on this compact complete-type set, keeping d_0 external moments. It retains at most R actual types. Each remains optimal at every owner history on its own recalled book; averaging wealth would not preserve that property. Owner-history independence of the private seed preserves the same type weights after owner deviations.

For each retained type and every GP history choose one attaining dual for each alternative conditional effort plan. Couple these types' financial kernels conditional on the common service schedule. Compress that joint post-effort law while retaining the moments in (30): its actual

prefix, liquidation and budget values; its selected conditional utility; every alternative plan's dual integrand; and all external owner, quote, source and dealer moments. Carathéodory gives the displayed bound. Selected utilities and recalled sources stay fixed, while preserved duals still bound every conditional alternative. Thus each selected type remains truly optimal at every owner history, including root-null histories. Support in the actual linked schedule polytope preserves every statewise cap. The compressed mark is not revealed early and therefore creates no new effort-contingent information.

The uniform finite representation, zero-weight atoms, compact amount bounds and continuous conditional values make the assessed-law set closed and compact. Refusal is feasible and the previously constructed young cash offer gives strictly positive utility. The continuous accepted objective therefore attains its maximum. Theorems D.1 and D.2 complete every other menu. Any accepted deviation is one of the feasible accepted pairs already compared by this maximum; a rejected proposal receives its actual refusal payoff. Hence the attaining pair is an optimal posted contract, not merely a best pair with undefined off-menu continuations. $\square$

D.3 Actual successful renewal from the selected full-history law

Proposition D.4 (A multi-history renewal source inequality). *Let $J(w) = V(w) - w \geq 0$. For an implemented young assessment, write N_H, N_L for actual expected high- and low-effort financed scales and $T \geq 0$ for nonnegative owner/current residual uses after cancelling internal transfers. At private type r and pre-effort history h , let ω_h be its history probability and α_h its actual viable posterior. Then*

$$W + \beta p \mathbb{E}[\mathbf{1}_H \omega_h \alpha_h (\mathbb{E}J(W_S) - \mathbb{E}J(W_F))] \geq \kappa N_H + (K - F_L) N_L + T, \quad (31)$$

$$\kappa = \frac{3}{500}, \quad K - F_L = \frac{259}{500}.$$

A positive active zero-wealth optimum consequently generates positive actual high-success mass at wealth with $J > 0$, and hence positive same-GP senior financing under the attained terminal policy.

Proof. At a selected high cell copy its entire contingent financial book and change effort only at that cell. Set

$$X_S = \delta W_S + \delta^2 Z_{2S}, \quad X_F = \delta W_F + \delta^2 Z_{2F},$$

where W includes the first-date market value of assignable claims and Z_2 includes only unassignable late service. Actual conditional IC gives

$$\alpha_h \Delta [EX_S - EX_F + \delta(\mathbb{E}J(W_S) - \mathbb{E}J(W_F))] \geq b n_h.$$

Current consumption cancels. Since $D_H = \alpha_h p (EX_S - EX_F) + \mathbb{E}X_F$, multiplying this IC by p/Δ yields

$$D_H \geq B_g n_h - \alpha_h p \delta (\mathbb{E}J(W_S) - \mathbb{E}J(W_F)) + \mathbb{E}X_F, \quad B_g = \frac{2}{5}.$$

Only dated consumption receives the actual priced/personal ratio β/δ . Telescope LP and dealer accounts using the selected plan-quantity law, add these conditional inequalities and then use $\alpha_h \leq 1$ in output coefficients. High output has value at most $F_H n_h$ and low output at most $F_L n_h$. Nonnegative failure consumption, financier rents and extra late-date discount cost can be dropped. This gives (31); no IC at an unobserved finer owner profile or pooled everywhere-low comparison was used.

At $W = 0$, a positive queue requires positive actual owner routing cash. Since $J(W_F) \geq 0$, (31) implies $\mathbb{E}[\mathbf{1}_H \omega_h \alpha_h p \mathbb{E}J(W_S)] > 0$. These weights are the actual high-success probabilities. At positive J , every attained senior optimum posts, since refusal gives only its wealth. The terminal source bound forces actual funded production whenever that posting has positive gain. Thus the successor wealth and repeat financing arise from the actual young optimum's own history law. $\square$

E The strict active interval for founder entry costs

The following comparison proves the full numerical interval used in Theorem 1.3; all continuations are optimized anew.

Write $V_u(W)$ for the terminal value, including refusal W and the fixed listing cost h , at outside application utility $u > 0$ and liquid wealth $W \in [0, 6]$. This is the value optimized in this cash institution, including all actual owner histories, technical forecasts and privately chosen books. Let U denote an active menu's gross GP utility before subtracting h . For its actual matched founder net utility r and positive queue q ,

$$r = \frac{u(q + \ell)}{c}, \quad m(q) = \frac{cq}{q + \ell}, \quad a = m(q)r = uq. \quad (32)$$

The already established full-history source inequalities give

$$U \leq L(W - a), \quad U - W \leq m(q)g - a, \quad L = \frac{200}{3}, \quad g = \frac{197}{500}. \quad (33)$$

In particular, a menu strictly improving refusal has $W > a$, $r < g < 1$, and $q < cg/u$. The same source and actual activation argument establish that V_u is nondecreasing and L -Lipschitz in wealth at each fixed u .

E.1 A payment change that preserves the actual information game

Let x_h be the founder's fixed treasury receipt on a matched terminal history and $c_F(h)$ the known nonpecuniary cost of its recorded tests. This includes a test deliberately taken before an off-path refusal. The actual common institution settles owner receipts at stage20, after technical orders and effort, privately to the treasury and recipient. The receipts remain unpledgeable before settlement. Only the specified publications and reports reach the GP and dealer. A change in the settlement formula therefore introduces no extra GP/dealer signal.

Lemma E.1 (All-history affine owner payoffs). *For $s \in (0, 1]$, either of the fixed receipt schedules*

$$x'_h = sx_h + (1 - s)c_F(h), \quad x''_h = sx_h + (1 - s)[1 + c_F(h)] \quad (34)$$

preserves every founder continuation best response and its consistent information assessment, while leaving the incumbent's payoffs unchanged. The resulting matched founder values are sr and $sr + (1 - s)$.

Proof. The founder's realized net payoff becomes, respectively, $s[x_h - c_F(h)]$ or $s[x_h - c_F(h)] + (1 - s)$. This is the same positive affine transformation at every terminal history in each of its perfect-recall information sets. It preserves all conditional comparisons, including publication, private reports, testing, consent and null histories. Incumbent receipts are unchanged, so its old best responses remain valid. The old owner strategy and belief system is therefore an assessment of the transformed game. Private closing settlement keeps the GP/dealer partition and all pre-effort information unchanged as well.

These are actual payments on all specified histories, including refusal; they are not a redefinition of a test cost. Since $c_F(h) \leq 1/1000$, each new receipt is a convex combination of the old admissible amount and a number at most $1.001 < 50$. The treasury reserves the actual conditional amount. No founder-to-incumbent liability is silently changed or netted. The common model's costs are nonpecuniary, so an earlier cash expense is not being financed by a later refund. $\square$

A monetary-test extension would require reimbursement at the actual cost-incidence prefix and a fixed feasible test-action tree. Scaling receipts and consumption then scales cumulative cash, but a reimbursement that newly enables a previously unaffordable test does not automatically preserve optimality. That extension is not used here.

E.2 Joint continuity without a continuous optimizer

Theorem E.2 (Terminal value as the outside option changes). *On any compact interval $[\underline{u}, \bar{u}] \subset (0, \infty)$, $V_u(W)$ is nonincreasing in u and obeys*

$$|V_u(W) - V_v(Z)| \leq L|W - Z| + \frac{Lcg}{\underline{u}}|u - v| \quad (W, Z \in [0, 6]). \quad (35)$$

Thus the actual terminal value is jointly continuous in wealth and the application outside option. No continuity of a maximizing policy is asserted.

Proof. For $0 < u' < u$, take an active assessed menu at u and apply the first transformation in (34) with $s = u'/u$. Its matched utility is $r' = sr$, so its *actual* induced queue is $cr'/u' - \ell = cr/u - \ell = q$. Matching, input, service, private GP books and technical forecasts are unchanged. The expected owner bill falls by $m(q)(r - r') = (u - u')q$. Treasury cost falls by that amount and both individual financier participation inequalities remain valid. The GP obtains its old terminal utility. Refusal is always available, proving monotonicity.

Now let $u' > u$ and choose a maximizing menu strictly improving refusal at u . First take a small enough step that $r' = (u'/u)r < 1$. The second transformation with $s = (1 - r')/(1 - r)$ preserves the owner assessment and makes its actual queue remain q at u' . Write $a' = u'q$. This increases the active expected owner bill by $a' - a$. The GP's service, actual input, liquid budget and technical comparisons remain unchanged; the additional fixed bill belongs to the treasury.

If $W > a'$, set

$$\lambda = \frac{W - a'}{W - a} \in (0, 1). \quad (36)$$

Draw the permitted independent activation coin after matching and before tests. Active matches run the transformed old program. Inactive matches perform no tests or investment and pay the founder its fixed r' from an actual reserved closing row. Its matched expected value remains r' , so the queue remains the one derived in (32).

Let B and EC denote the old discretionary GP budget and the expected mandatory current plan. Set $B' = \lambda B$. Copy the old active conditional cash claims and issuer liabilities; scale their ex-ante premiums, and scale the old unmatched uses in quantity. Actual conditional prices remain the old prices. The expected new mandatory bill is

$$EC' = \lambda EC + a' - \lambda a. \quad (37)$$

At the same original wealth W , the balanced treasury instruction gives

$$L' = EC' + B' - W = \lambda(EC + B - W) = \lambda L. \quad (38)$$

Actual retained LP output and dealer source/liabilities scale consistently. Both individual participation rows survive. The initial signed treasury transfer really delivers B' , and every active conditional bill remains reserved; (38) is not an outside cash injection.

The established full private-plan activation comparison applies: active books have their original conditional liquid source and prices, while inactive later bets are dominated for the terminal linear-consumption GP. Owner behavior and the GP/dealer information partition were preserved by Lemma E.1. Thus the constructed gross GP payoff is λU with genuinely optimal private effort and portfolio choices. The listing cost is paid once, giving

$$V_u(W) - V_{u'}(W) \leq (1 - \lambda)U \leq L(a' - a) = Lq(u' - u). \quad (39)$$

If $W \leq a'$, use refusal at u' instead: the same source bound gives $V_u(W) - V_{u'}(W) \leq U \leq L(W - a) \leq L(a' - a)$. No negative activation probability or unfunded call is needed. If refusal was optimal at u initially, monotonicity and refusal already give equality of the two values.

For every profitable compared menu $r < g$ and $q < cg/\underline{u}$. A step with $u'/u \leq 2$ has $r' < 2g < 1$, so the construction is available locally with a uniform loss constant. Partition any larger upward change into finitely many such steps and add (39). This yields $0 \leq V_u(W) - V_{u'}(W) \leq Lcg(u' - u)/\underline{u}$. Combining this bound with the already proved uniform wealth modulus establishes (35). $\square$

The proof is terminal: it does not delete a future productive option on inactive branches. A zero-wealth young GP lacks the positive buffer $W - uq$ used in (36). Uniform continuity of its actual future value now follows, but continuity of the young optimizing graph requires a separate argument for its binding incentive and participation conditions.

E.3 An explicit neighborhood with active zero-wealth finance

Corollary E.3 (A uniform strict young source). *For*

$$u_0 = \frac{117}{640000}, \quad |u - u_0| \leq 10^{-8},$$

the public treasury-paid-owner economy has an actual feasible zero-wealth young menu with strict conditional high-effort incentives and net entry value exceeding $3/80$. Its attained young maximum is therefore positive throughout this interval. The maximizing menu and successor law need not remain constant or vary continuously.

Proof. Keep the public two-test cash-owner benchmark at queue one, raw fill $1/4$, unconditional certificate probability $t = 3/32$, current GP budget and access zero, no late service and actual scale one on authentic funding histories. Use founder reward $4u$ and set

$$T(u) = \frac{1}{4} \left(\frac{3}{2000} + 4u \right), \quad \bar{z}(u) = \frac{t(317/500) - T(u)}{tp\beta}. \quad (40)$$

Let first-date success service be supported on $\{W_*, 2\}$, $W_* = 3363/640000$, with upper-point probability $\theta(u) = [\bar{z}(u) - W_*]/[2 - W_*]$. The actual LP source holds exactly. Throughout the displayed interval, $\theta \in (0, 1)$ and $\bar{z}'(u) = -50/3$. These actual payments preserve the benchmark owner assessment at its new rent. Zero liquid premium source and nonassignment of service exclude a privately funded hedge. No information-optimality claim is made for this feasible benchmark.

At u_0 the established continuation lower bound, conditional IC slack and net entry lower bound are

$$\frac{647909459}{638318500}, \quad \frac{9590959}{3191592500}, \quad \frac{9716088611}{255327400000},$$

respectively. On $[u_0/2, 2u_0]$, Theorem E.2 gives the outside-option modulus

$$\frac{Lcg}{u_0/2} = \frac{50432000}{351} < 143681.$$

Moreover $|\theta'(u)| < 9$ and the two actual support-point values belong to $[0, 3]$. The total expected continuation change is therefore smaller than $150000|u - u_0| \leq 3/2000$. The remaining conditional IC slack is at least

$$\frac{1381357}{510654800} > 0,$$

and net entry remains above $3/80$ by at least

$$\frac{507795779}{102130960000} > 0.$$

The comparison uses the true newly optimized V_u at each support point. It establishes a uniform feasible lower bound, not continuity of the maximum of all young menus. $\square$

Corollary E.4 (The same interval under minimum-output collateral). *The terminal continuity theorem and the preceding strict-source interval also hold after the actual public institution is reoptimized with the uniform service bound $z_d \leq (7/4)n$. The same interval can therefore be used in the stochastic public common economy.*

Proof. The payment and activation transformations copy every active n, z_d unchanged, so they preserve the smaller service covenant. The established source constants and terminal wealth modulus are unchanged. Thus Theorem E.2 applies to this reoptimized value. For the young benchmark use support $\{W_*, 7/4\}$ with the same $\bar{z}(u)$ in (40). At u_0 its upper probability is 624687/1116637. Now $|\theta'(u)| < 10$, while support-point values remain in $[0, 3]$. The conservative continuation modulus remains 150000. The established capped benchmark's IC and entry bounds, after the maximum continuation loss 3/2000, leave respectively

$$\frac{158989}{89330960} > 0, \quad \frac{289171279}{89330960000} > 0$$

as conditional IC slack and excess net entry above 3/80. Every continuation value is this smaller-cap economy's own optimized value; no former service law is substituted for its attained optimum. $\square$

F Minimum-output collateral and the active career benchmark

The service covenant and benchmark below are used in the general-process construction. The iid pair displayed here is a reference example; the main theorem replaces it by Assumption 1.1. The uniform creation-cost distribution in this reference calculation is replaced in the main theorem by the homogeneous positive cost k and its endogenous entry mixture.

F.1 The public stochastic primitive family

Use the final current-cash owner institution of Section A. Owners receive only fixed, nonnegative treasury-paid cash, with no owner liabilities, cross-owner obligations or discretionary owner-visible GP transfers. They may test, publish or suppress authentic evidence, report and consent at their actual finite histories. All signals reaching the GP reach its committed dealer. Owner receipts settle privately with the treasury after effort and create no additional GP/dealer signal. Clues arrive after routing. Test, creation and activation costs are nonpecuniary; actual owner reimbursements are consumed cash, while input and maintenance are goods.

At stage one, every original issuer credit and assignable old claim is settled or valued to establish actual spendable wealth $W \in [0, 6]$. No pre-offer consumption, hidden outside cash or unrecorded disposal is allowed. This is an explicit timing institution. A menu chooses discretionary budget $B \in [0, 50]$ and settles only the canonical initial directions

$$a = (W - B)_+, \quad f_0 = (B - W)_+, \quad W + f_0 - a = B.$$

Private consumption and current trading begin afterward from B . The treasury separately funds every chronological mandatory bill C_h . It does not substitute an expected cost for the cash needed in a particular history.

Retain the numerical effort, discount, testing and search primitives of the public common economy. In particular

$$p = \beta = \frac{4}{5}, \quad p_0 = \frac{2}{5}, \quad \delta = \frac{1}{2}, \quad b = \frac{1}{5}, \quad K = \frac{167}{100}, \\ c = \frac{1}{2}, \quad \ell = 1, \quad u_0 = \frac{117}{640000}, \quad h = \frac{1}{100000}.$$

Founder creation costs are uniformly distributed on $[u_0/4, u_0/2]$, independently of the later physical clue. The number u_0 parametrizes the active construction and these fixed costs; it is not imposed as the inactive economy's outside payoff.

Let the public iid states have

$$(A_L, A_H) = (1, 2), \quad (d_L, d_H) = (-1/4, 1/4).$$

Actual current amounts of x normalized units require $A_t x$ goods at their payment date. A successful venture pays $A_t(2 - d_t)n$ in U and $A_t(2 + d_t)n$ in E at each of its two original productive dates. The same original technical success mark controls both dates. GP dated utility is explicitly $\sum_k \delta^k C_{\tau+k}/A_{\tau+k}$; current retiring-owner cash utility is normalized likewise. Households retain ordinary log utility. The minimum-output service covenant is

$$0 \leq z_d \leq rn, \quad r = \frac{7}{4}, \quad d = 1, 2. \quad (41)$$

This is a uniform covenant in every aggregate state. It is not enlarged by favorable realized output. Technical purchased payments are bounded by $4n$ inside the same aggregate remaining-payment cap four as genuine nonproject claims. No technical event exists at $n = 0$. Unvested service is unassignable and unpledgeable; selling an old purchased claim leaves its original issuer liable at the original date.

Lemma F.1 (Reoptimized public careers and operating choices). *The public normalized reference with mean output $(2, 2)$ and (41) has an attained terminal value $V_{C,r}$, complete all-menu follower assessments and attained full-history young optima. Its value is nonlinear, nondecreasing and 200/3-Lipschitz on $[0, 6]$. A strictly incentive-compatible zero-cash young menu has net entry value greater than $3/80$, and an optimal young assessment generates positive same-GP finance after actual high-effort success.*

The same values and an optimizer can be obtained when a publicly verifiable operation $x \in [0, 1]$, chosen before the clue and private book, is available with

$$y(x) = \frac{7}{4} + \frac{x}{4}, \quad K(x) = \frac{333}{200} + \frac{x^2}{200}, \quad z_d \leq [y(x) - 1/4]n.$$

Both terminal and young optimal operations can be selected at $x = 1$.

Proof. Add the closed homogeneous row (41) to the complete conditional-book and finite-law programs in Section A. Conditional financial continuity uses the same actual likelihood bounds and continuously shrinking technical cap $4n$. Every unweighted GP optimality equality at every owner-null history remains imposed before compression. The finite owner game is independent of technical forecasts and private GP quantities. Its actual matched value R_F induces $q = (cR_F/u_0 - \ell)_+$. At that queue the treasury genuinely delivers B and every mandatory branch bill, with expected net cost

$$L(q) = \mathbb{E}_q C_h + B - W. \quad (42)$$

The actual queue changes cost and profits, not the delivered source. The same conditional forecast fixed point and individual financier participation therefore complete every menu. Losing proposals have funded counterfactual continuations, not undefined rejection payoffs.

The source bound used only the larger service cap two, so it remains valid with $r < 2$. At a technical-cap cell, normalize service and allocated actual dealer loss by $n > 0$; the same unit-scale bound applies, rather than an impermissible unscaled four-unit hedge. At $n = 0$ technical service and capacity vanish. Terminal nonproject dated consumption is dominated by current cash.

Increasing W while holding B and the continuation fixed changes only the reserved stage-one a, f_0 directions and lowers L . No saturated access row is enlarged. Rent-preserving activation

at $W' = \lambda W + (1 - \lambda)u_0q$ copies every conditional active claim and its issuer's row unchanged, sets $B' = \lambda B$, and adds the inactive founder receipt. It gives $\mathbb{E}C' = \lambda \mathbb{E}C + (1 - \lambda)u_0q$ and hence $L' = \lambda L$. The actual reverse comparison of full private plans is preserved by (41). Thus $V_{C,r}(0) = 0$ and its actual modulus remains $200/3$. The corresponding conditional young dual bound remains 125. Complete types and post-effort laws can therefore be compressed with every conditional source, alternative-plan, owner, quote and actual dealer moment retained. No pooled eleven-atom count is used. The positive optimizing sets stay away from canonical zero-queue refusal; all-menu Borel completion uses the same countable positive queue slices.

The public exhausted-source senior benchmark uses service $(1, 0)$ and no discretionary cash. It is feasible under $r > 1$ and gives

$$W_* = \frac{3363}{640000}, \quad V_{C,r}(W_*) \geq V_* = \frac{1781}{25000}.$$

Refusal gives $V_{C,r}(W) \geq W$; source and positive h make refusal optimal initially on $[0, 3h/197]$. Thus this value is nonlinear. At young queue one, use the actual public cash-owner benchmark, certificate probability $t = 3/32$, mean owner cash $T = 357/640000$, $B = 0$, no access fee and no late service. Conditional on success, let $Z_1 = r$ with probability $624687/1116637$, and $Z_1 = W_*$ otherwise. Then

$$\mathbb{E}Z_1 = \frac{12561}{12800}, \quad T + tp\beta\mathbb{E}Z_1 = t(F_H - K), \quad F_H = \frac{288}{125}. \quad (43)$$

The actual worst matched owner/input bill is $267557/160000$. The original fixed cash reimbursements preserve all voluntary owner decisions; no owner payoff depends on a GP order. At every authentic GP effort history, no cash or pledgeable service funds a side bet, and

$$\mathbb{E}V_{C,r}(Z_1) \geq \frac{141031096}{139579625} > 1, \quad (p - p_0)\delta\mathbb{E}V_{C,r}(Z_1) - b \geq \frac{1451471}{697898125} > 0.$$

Its net entry value is at least $4229816243/111663700000 > 3/80$. These are feasible lower bounds, not the assigned optimal law. They give a positive attained young optimum. Its actual high-success submeasure ν_{HS} satisfies the copied-book source bound

$$\beta \int (V_{C,r}(W) - W) d\nu_{HS} \geq T + \frac{3}{500}N_H + \frac{259}{500}N_L > 0. \quad (44)$$

The conditional comparisons use the GP's actual information cells. Since the actual senior option is bounded by $(197/500)$ times its expected financed scale, (44) gives positive same-GP successful repeat finance.

Finally, the physical posterior-floor lemma gives actual funded success probability $\pi \geq 2/35$. Replacing any operation $x < 1$ by one keeps the actual owner rewards, GP services, B , financial books and technical law unchanged and expands the service capacity. The extra LP receipts less extra real input are

$$n(1 - x) \left[\frac{\pi(36/25)}{4} - \frac{1 + x}{200} \right] \geq \frac{37}{3500}n(1 - x) > 0. \quad (45)$$

They fund the added input, with surplus retained by the same primary fund. No GP cash is created. First apply this comparison to all terminal menus, proving equality of the unrestricted operation value and the value at one. Then use that true terminal value in every young comparison. Its full continuation and conditional incentives are unchanged, so the same dominance proves selection at one for the young optimum. Strict GP preference is not asserted when the source improvement stays with the fund. $\square$

G Funded continuations at zero aggregate entry

Lemma G.1 (Actual funded venues at zero aggregate entry). *Fix a positive counterfactual venue mass v , the same potential founder population F and a creation-cost distribution G on $[k_{\min}, k_{\max}]$ with $k_{\min} > 0$. For every actual matched owner value $R_F \geq 0$, a unique counterfactual queue solves*

$$vq \in F[G(b(q)-), G(b(q))], \quad b(q) = \frac{cR_F}{q + \ell}, \quad 0 \leq q \leq F/v. \quad (46)$$

It is continuous in R_F for fixed $v > 0$. For a funded application, v must be an admissible positive portion of the existing conserved venue endowment with its named original-record reserves. In that domain the balanced treasury supplies a physically funded all-history follower assessment before financier participation is evaluated.

Proof. Creation cost is sunk and nonpecuniary, and the physical clue arrives only after routing. Hence the same matched owner assessment and consistent trembles can be selected across cost types without changing a continuation best reply. Types below $b(q)$ enter, those above refuse and an equal-cost atom may mix. The correspondence $(F/v)[G(b(q)-), G(b(q))]$ has nonempty compact convex values in $[0, F/v]$ and a closed graph: limits of cutoff fractions lie between the strict and weak probability cutoffs. The compact-domain fixed-point theorem gives a solution. For $R_F > 0$, b strictly decreases; any two different solutions would make strictly increasing vq nonincreasing. At $R_F = 0$ the positive cost floor gives the unique zero solution. The same joint closed-graph argument and uniqueness give continuity in R_F . The required fraction of a cutoff atom supplies a measurable mixing rule.

For the uniform calibrated G , this is simply $vq = FG(cR_F/(q + \ell))$. Its solution is zero when $cR_F/\ell \leq k_{\min}$; otherwise it stays below $cR_F/k_{\min} - \ell$ and tends to that bound as $v \downarrow 0$. No positive incumbent outside payoff is assigned to an inactive market. At the derived queue, the actual balanced treasury still supplies B and every mandatory conditional call. The completed conditional GP/forecast fixed point therefore applies at that queue. Founder entry is not computed using an old target label or division by zero. $\square$

H The physical posterior floor

Proposition H.1 (A posterior floor under arbitrary strategic garblings). *Let latent viability have prior $\alpha \in [\underline{\alpha}, \bar{\alpha}] \subset (0, 1)$. Conditional on viability and each α , a finite vector of good-only potential tests has all-null probability $\eta(\alpha) \geq \eta_0 > 0$. Nonviability produces only null results. Agents never observe viability directly. Every other signal, private coin and independent past-venture wealth is conditionally independent of viability given α and the complete physical test profile. Any actual history generated by measurable strategies using those observations satisfies, whenever it has positive probability,*

$$\Pr(\theta = 1 \mid \text{history}) \geq q_{\min} := \frac{\underline{\alpha}\eta_0}{1 - \underline{\alpha} + \underline{\alpha}\eta_0} > 0.$$

The bound covers strategic testing, messages, private reports, publication, consent and publicly observed committed financing. Consistent limiting beliefs at physically possible finite-alphabet histories satisfy the same bound.

Proof. Refine information to the mathematical profile consisting of α and the entire potential test vector. This refinement need not be known to any agent. A positive coordinate makes viability certain. At its all-null profile Bayes gives

$$\frac{\alpha\eta(\alpha)}{1 - \alpha + \alpha\eta(\alpha)} \geq q_{\min},$$

because this ratio increases in both arguments. Untaken tests and coarser observations average the same conditional probabilities.

No permitted strategy or independent randomization can distinguish viability within the all-null profile. Thus viability, the refined physical profile, and any actual generated history form a Markov chain in that order. Conditioning first on the profile proves that every actual posterior is a conditional average of numbers in $[q_{\min}, 1]$. This argument applies to arbitrary prior laws and measurable strategic garblings; it does not require that the actual history reveal the clue. Each full-support tremble in the stated finite tree obeys the same inequality. Its consistent posterior limits remain in the closed interval. Fabricated positives at physically impossible histories are not admissible observations. $\square$

Corollary H.2 (Technical quote bounds with a less-informed dealer). *Suppose positive primary scale and input are publicly and irrevocably committed before optional technical insurance. At every private profile, success conditional on viability lies in $[p_0, p] \subset (0, 1)$, including private effort mixtures. The dealer's actual technical-success quote then satisfies*

$$p_0 q_{\min} \leq \pi_D \leq p.$$

The GP may observe a finer garbling than the dealer. Its true-to-quoted technical likelihood ratios are at most

$$\frac{p}{p_0 q_{\min}} \quad \text{for success,} \quad \frac{1}{1-p} \quad \text{for failure.}$$

If dated personal and household weights are respectively δ^d and β^d , with $\delta \leq \beta$, the utility-to-price ratio at date d is bounded by $(\delta/\beta)^d$ times the corresponding likelihood bound.

Proof. Condition on viability and the complete profile. Every selected effort gives success at least p_0 conditional on viability. Taking the dealer's expectation and applying Proposition H.1 gives the lower quote bound; the upper bound is p . True success is at most p , true failure at most one, and quoted failure at least $1-p$. Multiplying these probability ratios by the actual dated utility-price weights proves the last assertion. Dealer participation must still use the quantities correlated with each true private effort and signal; the bound does not make the quote privately fair for every type. $\square$

For the two tests and clues used above,

$$\eta_0 = \frac{1}{4}, \quad q_{\min} = \frac{1}{7}, \quad \pi_D \geq \frac{2}{35}, \quad 1 - \pi_D \geq \frac{1}{5}.$$

The two likelihood bounds are fourteen and five, and the first-date utility-to-price bound is $35/4$. These bounds concern the already financed venture's actual technical outcome. A veto, absence of funding, or future discretionary GP acceptance is not its technical failure.

I Evidence and successful-vintage bounds

The common construction permits stronger conclusions than the existence of some informative finance. Its actual source places a lower bound on the scale financed with an informative posterior. A second bound fixes a nonempty interval of basic country endowments before an optimizing career is selected. Both arguments apply to every optimizing career mixture.

Theorem I.1 (An actual financed-scale bound for informative beliefs). *Consider any accepted young program in the public cash institution with liquid source W , actual financed-scale measure μ , total scale $N = \int d\mu$, and actual GP viability posterior $\alpha \leq \bar{\alpha}$. Suppose the known-viable productive present value under every effort is at most $F_H > 0$, and real input per unit scale is K . Let T be actual owner cash, C_0 current GP consumption, and P the household-priced personal claims generated by this young record. The latter includes purchased dated claims and any*

old-claim liquidation value under the actual pricing tower, but excludes output of a subsequently financed senior venture. For any $a < \bar{\alpha}$,

$$N_{>a} := \mu\{\alpha > a\} \geq \frac{(K - aF_H)N + T + C_0 + P - W}{(\bar{\alpha} - a)F_H}. \quad (47)$$

The inequality holds for arbitrary actual Borel history laws. A negative right side supplies no positive-mass conclusion.

In the zero-wealth public calibration, every accepted positive-queue program satisfies

$$N_{>3/5} \geq \frac{719}{2304}N + \frac{625}{576}(T + C_0 + P) > \frac{719}{2304}N, \quad (48)$$

$$N_{>835/1152} \geq \frac{500}{317}(T + C_0 + P) > 0. \quad (49)$$

Thus a strictly positive fraction exceeding 719/2304 of financed scale uses a GP belief above the ceiling available without new evidence.

Proof. Add the actual primary treasury, dealer and GP source rows, retaining the true private effort/order distribution in each liability. Current transfers and financial premiums cancel on their named accounts. Individual financier participation and nonnegative remaining uses imply

$$T + C_0 + P \leq W + \int (\alpha F_{\text{mix}} - K) d\mu, \quad F_{\text{mix}} \leq F_H.$$

With the optional operating field, operation one is selectable without being uniquely preferred. For any other chosen operation, the proved actual per-cell source improvement to one is at least $(37/3500)n(1 - x)$. Its net productive source is therefore bounded by the same $\alpha F_H(1) - K(1)$ envelope. This establishes the calibrated bounds for every optimizing operation, without separately replacing a smaller input cost by a larger one.

These are actual GP information cells, not hidden owner profiles at which an unavailable effort deviation is imposed. Splitting the posterior integral at a gives

$$\int \alpha d\mu \leq a(N - N_{>a}) + \bar{\alpha}N_{>a}.$$

Substitution and division by the positive denominator prove (47). The same integral argument covers non-finite laws.

In the calibration $F_H = 288/125$, $K = 167/100$, $\bar{\alpha} = 1$ and $W = 0$. Insert $a = 3/5$ to obtain (48). Insert $a = K/F_H = 835/1152$ to obtain (49). Actual matched founder value and routing give $T \geq uq > 0$. Before new evidence every physical profile has posterior at most 3/5; reports or garblings of those profiles cannot raise that ceiling. Consequently the excess-posterior scale requires acquired evidence used in the financing assessment. Informative suppression and private reports are included; publication frequency is not substituted for belief change. $\square$

Lemma I.2 (A uniform lower bound on successful productive scale). *In the public calibrated economy, suppose the attained young net value exceeds 3/80. Every optimal young assessment and every mixture of optimal careers then has actual young financed scale and common successful-vintage scale satisfying*

$$N_y > \frac{9}{6388}, \quad \frac{9\varepsilon}{55895} < f \leq 4\varepsilon. \quad (50)$$

Here f counts both original productive dates and both potential GP posting ages per country at birth mass ε .

Proof. The true terminal bound is $V(W) \leq (200/3)W$. Relative to the household price of the original record's claims, young successor utility therefore has ratio at most $(\delta/\beta)(200/3) = 125/3$.

Current consumption and old unassignable late service have smaller ratios. The consolidated zero-wealth source bounds gross young utility U by

$$U \leq \frac{125}{3}(F_H - K)N_y + bN_L \leq \frac{1597}{60}N_y.$$

This uses actual financed scale; replacing it by the larger raw match probability would give only the earlier queue bound. Since listing cost is positive and net value exceeds $3/80$, division gives $N_y > 9/6388$.

The physical posterior floor is $1/7$ at every actual GP cell and the smaller effort success probability is $2/5$. Private effort depends on the GP's information and book, not unobserved viability. Thus actual success probability per financed unit is at least $2/35$, even under a mixed private strategy. Young expected successful scale exceeds $9/111790$. Senior success is nonnegative and every original success pays at both dates, proving the strict lower bound on f . Two posting ages and unit scale give the upper bound. Integration preserves these bounds under every optimal-policy mixture; no continuous selector is required. $\square$