MANUSCRIPT EXTRACT

# Ownership and household preferences

T5 technical supplement

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Sections 50 and 53; original manuscript pages 270-275 and 288-292.

Section 50 proves extensions to concave indexed household utility u(C/A), identifies the boundary for unique consumption, and states curvature conditions for fully repriced evidence-removal comparisons.

Section 53 retains ordinary log households and permits general positive initial fund wealth. It derives portfolios and the exact international-account sign condition. A complementary nonhalf endowment family preserves net foreign assets across the active and inactive comparison.

These are separate stated extensions. General indexed preferences are not arbitrary utility of unindexed physical consumption. General initial ownership does not automatically preserve a trade deficit; the exact level condition is essential. No unrestricted combination of the two extensions is asserted here.

### How to read this supplement

This is a manuscript extract, not a self-contained proof note. The mathematical text, theorem numbers and printed cross-references are retained. References to sections and bibliography entries outside the extract point to the complete manuscript. Boundary-page content outside the selected sections is visually trimmed. The relevant theorem statements and their full local proofs are retained.

## 50 Household curvature and the scope of the common comparisons

The linear-household failure distinguishes equilibrium existence from identification of its consumption and international allocation. This section establishes the corresponding positive results for general concave household tastes. The utility primitive is explicitly

$$\mathbb{E}\sum_{t\geq 1}\beta^{t}u(C_{t}/A_{t}),\qquad \beta=4/5. \tag{445}$$

It is utility of consumption relative to contemporaneous productivity. Ordinary log utility is equivalent to its indexed version up to an exogenous term. Other ordinary utility functions in physical consumption cannot be normalized this way without changing preferences.

## 50.1 A general supporting-budget proof

**Theorem 50.1** (Concave household tastes in the same common economy)**.** *Retain the complete national-fund institution and all nonhousehold primitives of Theorem 45.1 or the bounded-positive process in Theorem 46.3. Let $u$ be strictly increasing and concave on $[0,\infty)$, finite on $(0,\infty)$, allowing $u(0)=-\infty$. The changed-preference economy has the same complete active common equilibrium with half-of-each national-fund holdings and its own supporting SDF $\beta A_t/A_{t+1}$. The actual financing and production law and the selected international accounts are unchanged. The same preference replacement also supports each completed inactive evidence or senior-eligibility comparison at its own basic-only allocation.*

*Proof.* Choose one of the actual attaining complete career mixtures in the indicated common theorem. Its derived normalized residuals are $c_U, c_E > 0$, $\overline{H} = c_U + c_E$, and $\bar{c} = \overline{H}/2$. The actual fund dividends are $A_t(c_U - fd_t)$ and $A_t(c_E + fd_t)$, with ex-dividend prices $4A_t c_c$. Households begin with half of each actual fund immediately after settled date-zero distributions. Thus their actual initial wealth is $W_0/A_0 = 4\bar{c}$, and utility begins at date one.

The fund pricing tower at $m_{0,t} = \beta^t A_0/A_t$ and the actual dated budgets of any nonnegative-wealth alternative imply

$$P := \mathbb{E}\sum_{t\geq 1} \beta^t C_t/A_t \leq 4\bar{c}. \tag{446}$$

No sufficiency of an abstract present-value allocation is assumed; this is a necessary inequality for actual feasible trading plans. A finite concave function on the open positive half-line has a nonempty finite superdifferential at every $\bar{c} > 0$. Any such supporting slope obeys

$$0 < \frac{u(2\bar{c}) - u(\bar{c})}{\bar{c}} \leq g \leq \frac{u(\bar{c}) - u(\bar{c}/2)}{\bar{c}/2} < \infty.$$

Thus positivity and finiteness follow from the utility primitives at every actual consumption level. Concavity gives $u(x) \leq u(\bar{c}) + g(x - \bar{c})$. This bounds the positive part of the discounted utility by an integrable affine function of consumption. Utility is therefore well-defined, possibly $-\infty$. With nonnegative tangent gap $G(x) = u(\bar{c}) + g(x - \bar{c}) - u(x)$, its exact extended-value expression is

$$\mathbb{E}\sum_{t\geq 1} \beta^t u(C_t/A_t) = 4u(\bar{c}) + g(P - 4\bar{c}) - \mathbb{E}\sum_{t\geq 1} \beta^t G(C_t/A_t) \leq 4u(\bar{c}). \tag{447}$$

There is no infinity-minus-infinity comparison. Half-of-each holdings consume $A_t\bar{c}$ and attain equality. Their normalized wealth is bounded, so their actual transversality condition holds. This proves global household optimality even against alternatives without a presumed transversality condition or an individual unit-supply cap. The positive supergradient supports the displayed household pricing kernel.

Every micro agent now faces exactly these own supporting household prices and unchanged preferences, controls and physical opportunities. Its already proved globally optimal information,

menu, private-book, effort, operation, entry and routing strategies remain optimal, including every funded off-menu assessment. The true successor wealth and biological law, original issuers, reserves and both venture payouts remain those actually constructed. Thus all markets and the invariant law close in the changed economy.

For either completed inactive intervention, use its actual basic-only residuals and $\bar{c} = (B_U + B_E - 2R)/2$ in the same budget proof. Its own prices are again $4A_t(B_c - R)$; the already completed treasury and rejecting financier continuations remain optimal at those prices. This proves existence in each changed-preference economy. It does not identify all household or active-equilibrium selections. □

## 50.2 An exact boundary for consumption identification

**Theorem 50.2** (The affine-contact boundary)**.** *At the common prices in Theorem 50.1, define*

$$S_g = \{x \geq 0 : u(x) = u(\bar{c}) + g(x - \bar{c})\}.$$

*An actual feasible household plan is optimal exactly when its present-value budget is exhausted and $C_t/A_t \in S_g$ almost surely at every date. If $\bar{c}$ is not in the interior of $S_g$, consumption is uniquely $A_t\bar{c}$. In particular, strict concavity suffices. Conditional injectivity of the actual two fund payoff columns then also identifies half-of-each quantities.*

*Conversely, if $u$ is affine on a neighborhood of $\bar{c}$ and $\mathbb{E}_\pi d^2 > 0$, the same active economy has distinct stationary optimal consumption allocations from the same initial wealth. Thus absence of an affine neighborhood is the exact consumption-uniqueness condition in this nondegenerate-risk family. The condition $\mathbb{E}_\pi d^2 > 0$ is automatic in EG and whenever the desired GM moment $\eta = \mathbb{E}_\pi(Ad)$ is positive.*

*Proof.* Equality in (447) requires both the nonnegative budget slack and every nonnegative discounted tangent gap to vanish. This proves necessity and sufficiency. The contact set is an interval. If $\bar{c}$ is not interior, it lies entirely on one side of $\bar{c}$; its present-value mean can equal $\bar{c}$ only if consumption is constant in normalized units. With unique consumption, subtracting the actual budget sequences of two optima gives the nonnegative-wealth martingale argument in Theorem 46.3. It identifies wealth without imposing an alternative transversality condition. If next-period $d$ is conditionally nonconstant, equality of the two actual fund payoff exposures identifies their quantities. This is injectivity, not spanning of an arbitrary aggregate state space. No such quantity uniqueness is asserted for redundant inactive funds.

For the converse choose a nonzero sufficiently small constant $z$ and let U hold

$$h_E = \frac{1}{2} + z, \qquad h_U = \frac{1}{2} - \frac{c_E}{c_U}z, \tag{448}$$

with E holding the complements. All holdings remain strictly inside their actual supplies. Their date-zero exchange is value balanced: $4A_0(c_U h_U + c_E h_E) = 4A_0\bar{c}$. It changes no settled consumption and adds no initial wealth. At every later date the actual fund dividends and unchanged holdings give

$$\frac{C_U}{A} = \bar{c} + f(1 + c_E/c_U)zd, \qquad \frac{C_E}{A} = \bar{c} - f(1 + c_E/c_U)zd. \tag{449}$$

Since $|d| \leq 1/4$, choose $z$ small enough that both quantities lie in the affine contact interval in every state. The conditional mean-zero tower for $d$ proves present-value equality. Bounded normalized wealth gives TVC, so both plans are optimal by the first part. They clear the actual funds and household goods, retain the same stationary productive law, and yield different consumption on positive probability because $f > 0$ and $\mathbb{E}_\pi d^2 > 0$. This is a stationary feasible exchange, not an unexplained initial wealth change. The full-affine case has the larger trade-sign failures proved in the preceding section. □

## 50.3 Curvature conditions for reoptimized comparisons

**Theorem 50.3** (Own-price comparison bounds under general curvature)**.** *Let $u$ additionally be strictly concave and continuously differentiable on $(0, \infty)$ with $u' > 0$. Define from the unchanged physical primitives*

$$\begin{aligned} H_- &= B_0 - C_0\epsilon, & H_+ &= B_0 + C_1\epsilon, \\ B_0 &= 199/25, & C_0 &= 160064, & C_1 &= 18, \\ \chi_u &= \frac{u'(H_-/2)}{u'(H_+/2)}. \end{aligned} \tag{450}$$

*For $0 < \epsilon \leq 10^{-6}$ the following statements hold for every bounded stationary candidate of the respective changed economy, including aggregate-history-dependent candidates.*

*If $\chi_u \leq 6/5$, removal of new evidence leaves only inactive real stationary allocations. If instead*

$$\frac{\beta}{\chi_u} > \delta, \qquad K - \chi_u F_H + \frac{\beta B_g}{\delta \chi_u} > 0, \qquad K - \chi_u F_L > 0, \tag{451}$$

*where $F_H = 288/125$, $F_L = 144/125$, and $B_g = 2/5$, removal only of new senior productive eligibility leaves only inactive real stationary allocations even with the information instruments available. In each case a completed inactive equilibrium exists by Theorem 50.1. These conclusions cover both EG and the bounded GM process under their original information, collateral and mark domains.*

*Proof.* The actual finite-path resource count, after all GP and owner consumption, gives $H_- \leq \widehat{H}_t := H_t/A_t \leq H_+$ in every candidate. Removing either access cannot enlarge this gross payment count. Identical household opportunities and initial half-fund wealth make the average of their two feasible plans feasible for each household. Its normalized consumption is in $[H_-/2, H_+/2]$, so its discounted value is finite. Optimal household values therefore cannot be $-\infty$. They are bounded above: normalized consumption in any clearing candidate is bounded by $H_+$, so positive utility is uniformly bounded. Strict concavity makes the average strictly better unless actual household consumption coincides at every date. Consequently each consumes $H_t/2$, without any spanning assumption.

The optimal average holds half of each fund and has strictly positive consumption. Its two-sided marginal valuation, at the original trading dates and with original paid collateral, supports the candidate's own normalized dated kernel

$$\widehat{m}_{t,t+j} = \beta^j \frac{u'(\widehat{H}_{t+j}/2)}{u'(\widehat{H}_t/2)}, \qquad \frac{\beta^j}{\chi_u} \leq \widehat{m}_{t,t+j} \leq \chi_u \beta^j. \tag{452}$$

The marginal valuation also applies directly to admitted actual funder cash flows; no unique extension of an incomplete-market price vector to arbitrary Arrow securities is inferred. At date zero no settled consumption is varied; stationarity supplies the same dated price law. The bounds in (452) have one whole-horizon factor, not a factor $\chi_u^j$.

For evidence removal, $\chi_u \leq 6/5$ gives exactly the bounds used in theorems 43.5 and 46.4. They dominate personal $\delta^j$. Apply the upper multiplier to actual nonnegative output before taking its conditional expectation. The original $Pd = 0$ condition and macro/physical independence give mean-two output at both original dates. The all-profile high and low source deficits are respectively 139/12500 and 8007/12500, including private benefit in the latter. Their operating-field minima occur at operation one, as already proved. Thus every terminal technical opportunity has negative source; its true optimized financial continuation is current wealth. Summing actual financier ledgers and all recalled private books then excludes every zero-wealth newborn issue. Finite original lives remove all productive stationary vintages.

For removal of senior productive eligibility, financial prices again dominate personal consumption prices by the first condition in (451). The true senior value is $W$, retaining old unassignable service separately and selling assignable old claims at their actual tower price once. Apply the all-information copied-book source in Corollary 36.5 with $\lambda = \beta/(\delta\chi_u)$ and output ceilings $\chi_u F_H, \chi_u F_L$. Its coefficients are positive by (451); operation one minimizes both. Newborn $W = 0$ therefore excludes positive finance even with useful evidence. No senior can originate a new venture, and finite lives give inactivity.

Existence in each case includes the same actual conserved-venue cutoff queue and balanced funded follower before individual financier rejection. Theorem 50.1 verifies household optimality at the resulting basic-only prices in the changed tastes. Strict concavity identifies its household consumption; inactive named holdings remain redundant. □

**Corollary 50.4** (Explicit indexed CRRA ranges)**.** *For $u_\gamma(x) = x^{1-\gamma}/(1-\gamma)$ when $\gamma > 0$, $\gamma \neq 1$, and $u_1(x) = \log x$, the selected active common equilibrium exists for every finite $\gamma > 0$. Throughout the original birth range $0 < \epsilon \leq 10^{-6}$, every $0 < \gamma \leq 8$ has the unique inactive real allocation after evidence removal. Every $0 < \gamma \leq 1/10$ has the unique inactive real allocation after removal only of new senior productive eligibility. These are explicit sufficient subsets of the general curvature conditions, not maximal bounds.*

*Proof.* Here $\chi_u = (H_+/H_-)^\gamma$. The largest original birth mass gives $r = 3980009/3899968 > 1$. Put $\chi_* = 200407/200000$. Exact integer-power inequalities give

$$r^8 < \frac{6}{5}, \qquad r < \chi_*^{10}.$$

Monotonicity therefore gives the first range. In the second, $\chi_u < \chi_*$. Both source coefficients and $\beta/\chi$ decrease with $\chi > 0$, so the source conditions follow from their positive values at that endpoint,

$$K - \chi_* F_H + \frac{\beta B_g}{\delta\chi_*} = \frac{3633943}{313135937500} > 0, \quad K - \chi_* F_L = \frac{201428}{390625} > 0.$$

Also $\beta/\chi_* > \delta$. Apply Theorem 50.3. The existence supergradient is positive and finite at every positive normalized consumption, so Theorem 50.1 covers all finite $\gamma > 0$. The exact

independent certificate checks both integer-power inequalities and the dated budget, tilt, pricing and source identities. □

# 53 Initial international ownership and derived household portfolios

The common construction need not endow both households with half of each national fund. This section retains ordinary log households and the actual fund-only income and trading domain. It changes only the primitive residence-by-fund endowment rows. GP wealth, corporate productive rights, funding obligations and the underlying national production technology are unchanged. The extension derives subsequent portfolios and the international level condition; it does not endogenize the original residence of those endowments.

## 53.1 Endowed positions, the initial exchange and stationary holdings

The timing of initialization is material. Date-zero dividends and consumption have already settled. Household $U$ is endowed with fractions $(a_U, a_E) \in [0,1]^2$ of the two remaining actual national funds, and household $E$ holds the complements. Exclude $(0,0)$ and $(1,1)$, which give one household zero wealth. Utility starts at date one. The households may then exchange these existing claims at their actual ex-dividend prices. The stationary portfolio law starts *after* that initial exchange; the primitive endowment fractions are not imposed again at later dates.

**Theorem 53.1** (General initial ownership in the same common economy)**.** *Retain the complete institution and other primitives of Theorem 45.1, or the bounded-positive process and common construction of Theorem 46.3. Use the same actual optimizing career mixture and population pair in both countries. Write its derived normalized national dividend coefficients as $c_U, c_E > 0$, with $H = c_U + c_E$ and $c_U - c_E = D$. At the new initial endowment rows, define*

$$\lambda = \frac{a_U c_U + a_E c_E}{H} \in (0,1). \tag{475}$$

*There is a common stationary equilibrium after the initial exchange in which household $U$ holds $\lambda$ of each national fund and consumes $\lambda A_t H$. Household $E$ holds the complements and consumes $(1-\lambda)A_t H$. The actual fund prices remain $4A_t c_c$, and both households' own marginal SDFs are $\beta A_t / A_{t+1}$.*

*The complete physical, information, contracting, entry and linked-career law is unchanged. Household consumption and financial wealth are unique at those prices and initial wealths. If next-period national risk is conditionally nonconstant, the actual two-fund payoff map is injective and the post-exchange quantities are also unique.*

*Proof.* The original dated accounts of the selected common mixture give actual fund dividends $A_t(c_U - f d_t)$ and $A_t(c_E + f d_t)$. The conditional mean-zero law of $d$ and the candidate household deflator price them at $V_{c,t} = 4A_t c_c$. Initial household $U$ wealth is therefore $4A_0(a_U c_U + a_E c_E) = 4A_0 \lambda H$. Its proposed initial exchange has zero value because

$$(\lambda - a_U)c_U + (\lambda - a_E)c_E = 0.$$

The other household takes the opposite positions. No new dividend, productive input, GP transfer or claim is created. The pre-exchange positions and post-exchange portfolio have the same actual total wealth.

The whole-market fund portfolio consumes $A_t H$ and satisfies the actual dated budgets. Multiplying its consumption, holdings and all homogeneous household cash prefixes by $\lambda$ gives a feasible individual plan. For any other individually feasible nonnegative-wealth plan, the positive deflator and sequential budget telescoping imply

$$\mathbb{E} \sum_{t \geq 1} \beta^t C_{U,t}/A_t \leq 4\lambda H. \tag{476}$$

This is a necessary budget inequality for actual trades; sufficiency of an abstract present-value allocation is not assumed. There is no additional individual unit-supply cap or presumed alternative transversality condition. The log supporting line at $\lambda H$ bounds normalized utility above by that of constant normalized consumption $\lambda H$. Bounded positive $A$ makes the subtracted exogenous term $\mathbb{E} \sum \beta^t \log A_t$ finite. The proposed plan attains equality. The same argument applies to $E$.

Strict log concavity identifies each consumption plan. Subtracting the actual budget sequences of another optimum and the proposed plan makes their deflated wealth difference a martingale. Nonnegative wealth and the proposed bounded normalized wealth give

$$\mathbb{E}\left|M_{0,t}(W_t - W_t^*)\right| \leq 2\mathbb{E}[M_{0,t} W_t^*] \longrightarrow 0.$$

Conditional Jensen identifies wealth at every earlier history. The two next total-payoff columns are the same actual columns as in GM, so their conditional injectivity identifies quantities when next $d$ is conditionally nonconstant. This requires no spanning of other aggregate risks. The proposed plan itself satisfies transversality directly.

Its actual household marginal ratios are $\beta A_t/A_{t+1}$ because the constant share and $H$ cancel. Consequently every micro contract, private-book comparison, individual financier participation row, reserve and off-menu continuation faces exactly the original own household prices. All previously proved optimizing strategies and actual source accounts remain valid. Their invariant linked population is unchanged. Here $\lambda$ depends on the constant $c_U, c_E$, not on the initial realization of $A$ or $d$. Thus the constant post-exchange holdings and the retained stationary physical process generate a stationary joint law. □

All household nonfinancial production and service income in this theorem arrives through the actual funds. Adding outside labor income, an individual nonhomogeneous borrowing restriction or nationality-specific claim access would require a different argument.

## 53.2 Unequal-wealth sharing before candidate prices are solved

**Lemma 53.2** (Proportional sharing at any admitted candidate)**.** *In the same fund-only household domain, condition on an initial state and its actual positive fund values. Let $W$ be total initial fund wealth and let household wealths be $\lambda W$ and $(1-\lambda)W$, with $0<\lambda<1$. At any bounded stationary candidate with positive aggregate household consumption $H_t^{\text{phys}}$, optimal clearing household plans satisfy*

$$C_{U,t}=\lambda H_t^{\text{phys}}, \qquad C_{E,t}=(1-\lambda)H_t^{\text{phys}}.$$

*Their own SDFs are therefore $\beta^j H_t^{\text{phys}}/H_{t+j}^{\text{phys}}$. The same-public no-evidence and removed-senior-access all-candidate source exclusions consequently extend to these initial ownership rows, on their original respective parameter domains. Their completed inactive equilibria use their own basic-only prices and recomputed wealth shares.*

*Proof.* Actual household budgets, holdings and cash prefixes are positively homogeneous in initial wealth and the plan, because income is entirely fund income and the individual constraints are homogeneous. Normalize the two candidate plans by $1/\lambda$ and $1/(1-\lambda)$. Each is feasible from $W$. Their weighted average is the whole-market plan, whose consumption is $H_t^{\text{phys}}$ and whose holdings are the total actual supply. Its two proportional pieces are thus feasible alternatives to the original households. Joint clearing may bound actual holdings by supply; it does not impose that bound on unilateral normalized deviations.

Put $X_t=C_{U,t}/\lambda$ and $Y_t=C_{E,t}/(1-\lambda)$. Individual optimality against the proportional alternatives implies

$$\mathbb{E}\sum_{t\geq 1}\beta^t[\lambda\log X_t+(1-\lambda)\log Y_t]\geq \mathbb{E}\sum_{t\geq 1}\beta^t\log H_t^{\text{phys}}.$$

Weighted strict log concavity gives the opposite inequality because $\lambda X_t+(1-\lambda)Y_t=H_t^{\text{phys}}$. The proportional alternatives have finite value on the bounded positive resource band. Each candidate household consumes no more than aggregate resources, so utility has bounded positive part, and optimality rules out value $-\infty$. Hence the inequalities are well-defined and equality forces $X_t=Y_t$ almost surely at every date. The share is fixed along future histories conditional on the initial state, so it cancels from the marginal ratios.

The original actual resource band now yields precisely the same whole-horizon kernel bound used in Theorems 46.4 and 48.3. Their actual-cell effort comparisons and consolidated LP/dealer source accounts do not depend on which household owns the financed fund. Their all-candidate exclusions therefore apply, with no enlargement of the original removal parameter interval.

For inactive existence apply the construction in Theorem 53.1 to $c_c^0=B_c-R_c$ and recompute $\lambda^0=(a_Uc_U^0+a_Ec_E^0)/(c_U^0+c_E^0)$. The existing balanced treasury, induced zero-entry cost-cutoff queues, complete funded follower assessments and simultaneous individual financier rejection remain feasible and optimal at those actual basic-only prices. Thus existence uses those completed continuations, not merely a negative aggregate source. Consumption and wealth are identified; the inactive rank-one portfolio quantities can remain redundant. □

At arbitrary candidate prices the initial share can depend on the initial public state.

Conditioning on that state is sufficient for the last proof. It does not itself construct an invariant joint law of later public states and that initial share. The constructed active and inactive equilibria use constant normalized fund coefficients, which supply the stationary post-exchange initialization separately.

### 53.3 The exact international level condition

**Proposition 53.3** (Initial ownership and selected external accounts)**.** *In the selected common equilibrium define*

$$\delta_c = c_U - \lambda H = (1 - a_U)c_U - a_E c_E, \qquad \mu = \mathbb{E}A, \quad \eta = \mathbb{E}(Ad) > 0.$$

*The actual external accounts are*

$$\begin{aligned} NFA_{U,t} &= -4A_t\delta_c, & TB_{U,t} &= A_t(\delta_c - fd_t), \\ \mathbb{E}TB_U &= \mu\delta_c - f\eta, & \mathbb{E}RD_{\text{net}} &= f\eta - \mu\delta_c, \\ \mathbb{E}\mathcal{A}_U &= \lambda\mu v_E, & \mathbb{E}\mathcal{L}_U &= (1-\lambda)\mu v_U, \\ \bar{r}_A &= \tfrac{1}{4} + \frac{f\eta}{\mu v_E}, & \bar{r}_L &= \tfrac{1}{4} - \frac{f\eta}{\mu v_U} > 0. \end{aligned} \tag{477}$$

*Both foreign legs are positive. This selected equilibrium is a debtor with a trade deficit and positive net foreign return dollars exactly when*

$$0 < \delta_c < \frac{f\eta}{\mu}. \tag{478}$$

*For the fixed primitive rows $a_U = a_E = 1/4$, the active common equilibrium instead has a strict debtor trade surplus throughout the original $\epsilon \le 10^{-6}$ range, despite both positive ordered gross rates.*

*Proof.* Subtract actual foreign stock values:

$$\lambda A_t v_E - (1-\lambda)A_t v_U = -4A_t\delta_c.$$

Domestic net output available to households is $A_t(c_U - fd_t)$ and resident consumption is $\lambda A_t H$, giving the trade balance. Each return dollar is its actual next coupon plus next ex-dividend value minus its old value. With $RD_{\text{net},t+1}$ measured over $(t, t+1]$, those actual terms give $NFA_{t+1} - NFA_t = TB_{t+1} + RD_{\text{net},t+1}$ before any expectation. Stationarity sets expected old-value changes to zero and gives the displayed mean accounts. Original productive issuers, both payouts, survivor values and new subscriptions are kept as in the GM ledger; new issues are not capital gains. Dividing each foreign mean return by its mean beginning stock cancels its constant positive ownership fraction and gives the two rates. Actual positive national dividends imply $\bar{r}_L > 0$. The three desired signs are therefore equivalent to (478).

For the specified quarter rows, $\lambda = 1/4$ and $\delta_c = H/4 + D/2$. The actual common resource bound gives $H \ge 199/25 - 160064\epsilon > 7.79$, while $f \le 4\epsilon$ and $\eta/\mu \le 1/4$ imply $f\eta/\mu \le \epsilon$. Hence $\delta_c > f\eta/\mu$ for every actual optimizing mixture in the stated mass range. This is a primitive ownership change in the solved common economy, not a proposed portfolio at incompatible

prices. Ordered gross returns do not supply the missing level condition. □

## 53.4 A primitive nonhalf family with unchanged NFA across interventions

**Corollary 53.4** (Asymmetric initial ownership and invariant net assets)**.** *Fix $a_E = a$, $a_U = 1-a$ with $0 < a \leq 1$ before micro optimization. Choose the independent basic gap in the nonempty interval*

$$0 < D < \frac{9\epsilon}{55895}\frac{\eta}{\mu a}, \tag{479}$$

*intersected with the already required common-economy and reserve bounds. For every common selected optimizing career mixture,*

$$\lambda = \frac{1}{2} + \frac{(1-2a)D}{2H}, \qquad \delta_c = aD. \tag{480}$$

*The derived holdings are generally not half of each fund. The equilibrium has $NFA_{U,t} = -4A_t aD$, negative mean trade balance and positive net foreign return dollars.*

*In the completed no-evidence and senior-access-removal comparisons at the same initial primitive rows, their own recomputed shares satisfy $\delta_c^0 = aD$. Statewise NFA is therefore unchanged, while mean trade balance becomes $\mu aD > 0$. All-candidate inactivity retains the respective parameter qualifications of the underlying comparisons.*

*Proof.* Substitute the complementary initial rows into (475); $c_U - c_E = D$ gives (480). The uniform actual success-scale bound $f > 9\epsilon/55895$ then implies $0 < aD < f\eta/\mu$ from the primitive gap before an optimizing mixture is selected. The prior strict reserve and population bounds can be intersected with this positive interval without making it empty. Equation (477) proves the active accounts.

In each completed inactive comparison the national residual difference is still $D$, though their sum and the actual initial fund values change. Recompute the household share at those prices. The same algebra gives $\delta_c^0 = aD$, and the inactive $f = 0$ gives the stated surplus and NFA. The actual optimal inactive wealth line is scaled to this new share; gross return rankings need not be unique on redundant fund positions. At $a = 1$ both initial household wealths remain positive, so the argument still applies. At $a = 0$ existence also holds but $NFA = 0$, excluding the strict debtor conclusion. No GP residence choice, new personal wealth or unchanged optimal portfolio fraction across interventions is assumed. □