MANUSCRIPT EXTRACT

Information, directed offers and entry

T4 technical supplement

Elies Lucio and Hugues Pouget

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The fully funded information game yields an attainable matched-rent frontier for the entire admitted continuous financial menu. Conserved physical venue capacity then derives applications, the global optimal effective offer, and positive costly participant entry.

The extract includes Lemma 58.1, Theorem 58.2, Corollary 58.3, their complete local proofs, the positive-size venue foundation, and a stationary household closure. Its square-loss identity rules out every admitted continuous-menu deviation; the square-root formula is one consequence.

The clue arrives after routing and originators retire at current closing. Worst-case financial reserves remain required for every slot. Early informed routing, endogenous location sorting, active programme choice and private successor wealth are outside this construction.

How to read this supplement

This is a manuscript extract, not a self-contained proof note. The mathematical text, theorem numbers and printed cross-references are retained. References to sections and bibliography entries outside the extract point to the complete manuscript. Boundary-page content outside the selected sections is visually trimmed. The relevant theorem statements and their full local proofs are retained.

58 Directed offers with conserved matching capacity

A separate market for every continuously chosen offer does not inherit the usual competitive-search interpretation from the original bounded matching formula. This section proves the obstruction and supplies an explicit replacement. The replacement counts physical venue capacity, derives applications and profitable deviations from actual positive-size venues, and optimizes the entire insured-information menu. It also permits positive-cost founder creation and GP entry. The private clue arrives after routing; originators still retire at current closing. These restrictions remain part of the result.

58.1 Why exact-menu market scale matters

Write E for applicant mass and V for funded GP slots. Under $M(E,V) = \chi EV/[(1+E)(1+V)]$, a new venue with slot mass ε offers an applicant meeting probability bounded by

$$p_A(E,\varepsilon) = \frac{\chi\varepsilon}{(1+E)(1+\varepsilon)} \leq \frac{\chi\varepsilon}{1+\varepsilon}. \tag{513}$$

Suppose feasible matched incremental founder utility is bounded above by $\overline{a}$. If an alternative route gives a strictly positive incremental value u_0, no founder applies to the new venue once the right side of (513) times $\overline{a}$ is below u_0. The conclusion is uniform in its queue and contractual details. A positive application cost can supply the same strict inequality. A positive *absolute* unmatched continuation alone cannot: that continuation cancels when comparing routes.

If positive incremental reservation values hold only almost everywhere, the applicant mass still vanishes as $\varepsilon \downarrow 0$ in a finite population. Its indicator is bounded by $\mathbf{1}_{\{u_x \leq \chi\overline{a}\varepsilon/(1+\varepsilon)\}}$; dominated convergence applies. The GP meeting probability is then at most χ times that vanishing applicant mass. Avoiding this conclusion with rewards diverging as $1/\varepsilon$ requires actual diverging per-slot collateral, which the bounded productive claims and reserves do not supply.

Consequently a single atomless GP posting a previously unused exact menu can attract no applications even when that menu would be profitable at positive market scale. Positive-size existing venues may therefore support many different menus: the posting deviation is blocked by market creation, not by a proved optimal division of surplus. This does not make every arbitrary menu an equilibrium; existing entry and participation must still hold. It establishes that the continuous empty-market probabilities alone do not prove competitive-search optimality. Under a literal exact-label interpretation, diffuse menu distributions also produce no positive-mass venues and hence no aggregate meetings.

Dividing the same participants equally among n venues gives

$$nM(E/n, V/n) = \frac{\chi nEV}{(n+E)(n+V)}. \tag{514}$$

Its derivative with respect to continuous $n > 0$ has the sign of $EV - n^2$, and its limit as $n \to \infty$ is zero. For $E = V = 1/2$, one venue yields $1/9$ matches and two yield $2/25$. Thus menu labels cannot create new copies of the constants 1 without changing the physical technology. Moreover, positive matching rates along shrinking balanced markets are incompatible with a jointly continuous zero extension along the empty axes. A valid architecture must specify which normalization it uses.

58.2 A third physical input and derived application queues

Let a venue have conserved capacity $a > 0$, and apply the original local formula per unit of that capacity:

$$\widehat{M}(E, V, a) = aM(E/a, V/a) = \frac{\chi aEV}{(a+E)(a+V)}. \tag{515}$$

This technology is constant returns in its three physical inputs and satisfies $\widehat{M} \leq \min\{E, V\}$. Dividing all three inputs proportionally preserves total matching. Each potential one-slot GP has $\ell > 0$ nontradable, location-specific venue-capacity units; entry activates them at the GP's stated nonpecuniary cost. Hence $a_m = \ell V_m$, and reposting or splitting a menu does not create capacity. The separate real maintenance input of basic production is not charged again here. Founders' opportunity locations and GP venue locations are fixed; offers compete within each country. Household claims may be held internationally. Cross-country venue relocation and endogenous residence sorting require additional optimization and are not included.

Set $c = \chi\ell/(1+\ell)$ and $q = E/V$. Substitution gives

$$\widehat{M}(E, V, \ell V) = \frac{cEV}{E + \ell V}, \qquad p(q) = \frac{c}{q+\ell}, \qquad \lambda(q) = \frac{cq}{q+\ell} = qp(q). \tag{516}$$

The two side probabilities are at most one. The associated common matching measure has dominated applicant and GP marginals, so each founder makes one application and each funded slot obtains at most one venture. Every slot must also reserve its full worst-case financial call; matching capacity is not cash and cannot replace that reservation.

Entry and application precede the privately shared clue. Entry costs are independent of that later information, and all applicants therefore have the same conditional matched prior at a given public state and location. GPs commit their menus before applications. After matching, the actual finite test, publication, consent and insured-settlement game is the one proved in the preceding section, including its fair quotation and measurable off-menu assessment completion. No other financier replaces the committed offer after evidence. Allowing founders to know their clue before routing would permit endogenous type selection and invalidate the scalar reduction below unless that selection were separately solved.

Lemma 58.1 (Actual matched-rent frontier)**.** *Let Q be the household value of the full actual viable productive claim, K its current primary input, and κ the union-certificate probability.*

Suppose

$$\overline{\alpha}Q < K < Q, \qquad \underline{\alpha}\eta_j(1-\eta_k)(Q-K) > c_j \quad (j \neq k).$$

Put $W = \kappa(Q-K) - c_F - c_I > 0$. *For every feasible continuous menu and consistent participating assessment, founder matched utility* r *net of its test cost and GP matched cash* g *satisfy* $r+g \leq W$. *Every pair* $(r, W-r)$ *with* $0 \leq r \leq W$ *is attained by an actual insured menu.*

Proof. Consolidate all current transfers, liquidated claims and LP calls. Actual output and LP participation bound the owners' total current net cash by $\mathbb{E}[(\theta Q - K)x]$. Incumbent participation covers its actual test cost. Conditioning on the full private signal profile, a certificate makes financing efficient and joint absence makes it inefficient. The displayed marginal-information inequalities make both tests maximize this surplus at each clue. Subtracting both true test costs gives $r + g \leq W$. This bound covers the complete admitted report, claim, hedge and lottery domain, not only a cash quote or a public no-publication posterior.

To attain it, on histories with both observed tests and required consents pay founder $c_F + r$, incumbent c_I and GP $W - r$, and finance exactly when an authentic certificate is published. On skipped-test and refusal histories disable finance and refund the unused reserve. The founder strictly prefers its reimbursed test when $r > 0$; the incumbent's test is weakly optimal. Select authentic publication, which does not change the fixed receipts, and consent, which preserves them. Matched founder utility is r at each clue. The actual reserve is $R^{LP} = K + \kappa(Q-K)$, and an activated call is $K\mathbf{1}_{\{\text{certificate}\}} + \kappa(Q-K)$. Its expected call is κQ, the value of the actual output claim. Failed-test fees are paid from this earlier advance. Rent division therefore changes neither the total fee nor its existing source. □

Let $u > 0$ be the best gross application utility before deducting creation cost. A used menu with matched rent r must give $rp(q) = u$. Thus routing implies the explicit queue and fill rules

$$q(r,u) = \left(\frac{cr}{u} - \ell\right)_+, \qquad \lambda(q(r,u)) = \left(c - \frac{\ell u}{r}\right)_+. \tag{517}$$

If $cr/\ell \leq u$, the offer attracts no applications. At $r = 0$ its queue and fill are zero for $u > 0$.

These formulas have an actual positive-venue foundation. Let μ be a finite measure of existing slots with a positive rent on a positive mass and relevant rents $0 \leq r \leq W$. The source frontier gives this bound for nonnegative incremental GP rewards; any own-cash offer with $r > W$ has a negative incremental reward and is dominated by no activity and retaining outside cash. This bound ensures a finite first moment and continuity of the integral below; finite slot mass alone would not suffice for unbounded rewards. Let founder supply be $AG(u)$ for a continuous creation-cost CDF G positive at positive arguments. The exact clearing equation is

$$AG(u) = \int q(r,u)\,\mu(dr). \tag{518}$$

The left side is nondecreasing and bounded; the right side is continuous and strictly decreasing while positive, diverges at zero and vanishes above all useful offers. There is a unique positive root. Adding a new venue of actual size ε and capacity $\ell\varepsilon$ adds $\varepsilon q(r,u)$ to its right side. Strict crossing and continuity imply $u_\varepsilon \to u$, while its actual applicant mass is $\varepsilon q(r, u_\varepsilon)$ and its GP fill tends to (517). Thus the small-venue deviation is derived from feasible positive markets. It is

part of the directed competitive institution, not an arbitrary conditional probability assigned to a null exact label. This differs substantively from (513).

58.3 Global posting and costly participant entry

Theorem 58.2 (Optimal continuous directed offers)**.** *For $0 < u < cW/\ell$, every continuous matched-menu deviation is bounded by*

$$\Pi(r;u) = \left(c - \frac{\ell u}{r}\right)_+ (W - r), \qquad 0 \le r \le W.$$

The unique maximizing effective founder rent, maximal gross GP entry reward and resulting queue are

$$r^*(u) = \sqrt{\frac{\ell u W}{c}}, \qquad \Pi^*(u) = (\sqrt{cW} - \sqrt{\ell u})^2, \qquad q^*(u) = \sqrt{\frac{c\ell W}{u}} - \ell. \tag{519}$$

The insured menu of Lemma 58.1 attains the optimum over the entire admitted continuous financial domain.

Proof. Routing gives (517); the full-menu source frontier gives $g \le W - r$. In the positive-queue region, $\Pi = cW + \ell u - cr - \ell u W/r$. With the displayed r^*,

$$\Pi^*(u) - \Pi(r;u) = \frac{c(r - r^*)^2}{r} \ge 0. \tag{520}$$

In the no-queue region gross profit is zero, strictly below Π^*. The efficient insured rent split supplies equality at r^*, proving a global deviation result rather than only a first-order condition. □

Let potential founder and GP masses be $A, F > 0$, with continuous CDFs G, J of their nonpecuniary creation and venue-activation costs. Entry costs are paid in utility, not by inventing cash for a zero-cash agent. Suppose $G(0) = J(0) = 0$ and both CDFs are positive at every positive argument. Entry and application clearing reduce to

$$AG(u) = FJ(\Pi^*(u))q^*(u), \qquad 0 < u < cW/\ell. \tag{521}$$

The left side is nondecreasing and the right side strictly decreasing while positive. At zero the right side diverges; at the upper endpoint it vanishes while the left side is positive. Thus there is exactly one root, giving actual entry $E = AG(u)$ and $V = FJ(\Pi^*(u))$. Each individual follows its actual cost threshold; the indifferent boundary has zero mass. Different mechanisms attaining the same rent pair need not be unique. Positive lower cost supports are also permitted when the corresponding strict crossing conditions hold.

Corollary 58.3 (Strict entry at bounded population caps)**.** *Let $q_0 = A/F$ and define*

$$u_0 = \frac{c\ell W}{(q_0 + \ell)^2}, \qquad r_0 = \frac{\ell W}{q_0 + \ell}, \qquad \Pi_0 = \frac{cW q_0^2}{(q_0 + \ell)^2}. \tag{522}$$

If every founder cost is strictly below u_0 and every GP cost strictly below Π_0, all potential participants strictly enter. Their actual queues, globally optimal menus and entry payoffs are those displayed. With $A = F = 1/2$ and $\chi = \ell = 1$, they reduce to $q_0 = 1$, $r_0 = W/2$, $u_0 = \Pi_0 = W/8$ and matched mass $1/8$. The inequalities may hold in every public state despite state-dependent W, generating the same constant cohort mass from optimizing behavior.

Proof. Substitute q_0 into (519) and the routing indifference equation. The resulting values are (522). The strict cost inequalities make both caps individual best responses; (520) excludes every alternative continuous menu and (518) verifies the applications and new-venue deviation rule. □

58.4 A primitive stationary household and directed-search economy

The interior-entry result can be closed without supplying a price or a stationary queue. Let a viable venture pay the constant actual amounts $y_d \geq 0$ at ages $d = 1, \ldots, J$, with at least one positive payment, and let $\beta \in (0, 1)$. Define

$$Y = \sum_d y_d, \qquad Q = \sum_d \beta^d y_d.$$

Take constant basic output $B > R$, the information inequalities of Lemma 58.1, the finite potential populations and cost CDFs just specified, and the conservative primitive reserve bound $2F[K + \kappa(Q - K)] < B - R$ for two identical countries. Solve (521); it gives E, V, the optimal menu and actual world financed mass $b = 2\kappa V \lambda(q^*)$. Hidden balanced fresh cohorts realize the actual individual likelihoods and aggregate counts; no extra cross-match private signal is released. Every funded project is viable and pays each y_d at its true age.

Household consumption and nonhousehold fee consumption are

$$H = B - R + b(Y - Q) > 0, \qquad G_{\text{nonHH}} = b(Q - K). \tag{523}$$

Actual investment, fees and consumption satisfy $H + bK + G_{\text{nonHH}} + R = B + bY$. The same proportional household ownership of actual initial vintages and new pooled subscriptions gives the budget $H + bQ = B - R + bY$. Constant consumption makes the log marginal-utility deflator β^d, which values the actual productive claim at exactly the displayed Q. Conversely these budgets, fair prices, log concavity and discounted budget telescoping prove household global optimality. Bounded finite-age claims imply transversality. Actual constant cohort aging gives the full stationary marked population. Thus prices, information, continuous posting, directed applications, positive-cost creation and GP entry, primary financing, actual production and household portfolios belong to one primitive stationary economy.

For an exact interior example, take $\beta = 4/5$, $y_1 = y_2 = 2$, $K = 5/2$, $B = 100$, $R = 1/50$, $\chi = \ell = 1$, $A = F = 1/2$, and both creation/activation costs uniform on $[0, 1/20]$. Use $\kappa = 3/8$, $c_F = 1/1000$ and $c_I = 1/2000$. Then

$$Q = \tfrac{72}{25}, \quad W = \tfrac{141}{1000}, \quad r^* = \tfrac{141}{2000}, \quad u = \Pi^* = \tfrac{141}{8000}, \quad q = 1,$$

$$E = V = \tfrac{141}{800}, \quad M_{\text{country}} = \tfrac{141}{3200}, \quad b = \tfrac{423}{12800}, \quad R^{LP} = \tfrac{1057}{400}, \quad H = \tfrac{8001361}{80000}.$$

Both entry thresholds are strictly interior. The rational verification artifact checks the resource equations and the exact global deviation certificate; it does not use numerical success as an existence argument.

The asymmetric economy can instead use Corollary 58.3 in every country and public state. With the same half-unit population caps, each country's optimized matching mass becomes $1/8$ and certified birth mass $3/64$. Its household prices must be recomputed at those actual cohorts, rather than copied from the random-pool calculation. For $B = (4, 11/2)$, $K_U = K_E = 11/5$, $y_U = (4, 4)$ and $y_E = (2, 6)$, each paid in halves at the two productive dates, the independent inverse calculation gives

$$H = \left(\tfrac{15377737}{3807200}, \tfrac{15377737}{2727200}\right),$$

$$Q_U = \left(\tfrac{294048}{118975}, \tfrac{294048}{85225}\right), \qquad Q_E = \left(\tfrac{269748}{118975}, \tfrac{269748}{85225}\right).$$

At $W_{cz} = \kappa(Q_{cz} - K_c) - c_F - c_I$, the smallest entry reward is $225813/76144000 > 1/1000$. Founder and GP costs in $[1/2000, 1/1000]$ therefore induce strict all-entry choices, actual queues $q = 1$ and optimal rent splits $W_{cz}/2$. Both national insurance reserves and all information conditions hold at these same prices. The accompanying asymmetric external accounts use these exact cohort masses. This is a common information, directed-search and household-portfolio construction; early informed routing, endogenous location sorting, active program choice and private successor wealth remain outside its declared domain.