For a Fistful of Gold

Elies Lucio *

1 November 2026

Abstract

By empowering conflicts, deforestation and intimidation of the legal economy, illegal mining of gold constitutes a challenging puzzle for policymakers. Because provenance is only a claim, it enters the global market avoiding regulation. In this paper, I study how trade resists policy efforts to stop it and I develop a blockchain-based certification aimed at hindering it. To capture its resilience, I model a coerced jungle economy in which an armed group controls the sale of both the inputs and outputs. The trade can power through a policy shock because households mine to meet basic needs and the armed group can absorb lower margins without cutting output. The intermediary can recover margins both by adjusting prices in the jungle and by re-allocating the metal via its selling mechanism. I model the re-allocation by looking at the receipts following the policy-driven demand shock. These generate a price effect on regulated bids which transmits imperfectly, and up to a ceiling, to receipts due to laundering and the level of competing bids. Blockchain breaks the cycle by recording provenance from a minting node onward; with physical custody, certified gold cannot be relabeled downstream, so leakage is confined to the mine. Leakage at the mine level is looked at through the lens of contracting and auditing which determine the blockchain's credibility. At the macro level, the blockchain segmentation breaks the pooling of innovation, and temporary public support can set certified research on a self-sustaining path. At the micro level, the royalty of a legal mine grows with the certified frontier, changing the incentive of affected countries to take back control of their jungle. Macro and micro work as scissors: protection payments can rise, but they fall relative to the value of a legal mine, and governments that can fund repeated attempts eventually intervene.

Keywords: illegal mining; intermediation; mechanism design; provenance; directed technical change; state capacity; conservation.

*Université Paris-Dauphine–PSL. The author thanks Gilles Chemla, Joseph Kaboski, Philippe Aghion, Simon Johnson, Lars Peter Hansen, Fernando Alvarez, Catherine Casamatta, Quentin Vandeweyer, Jerome Dugast, Olivier Tercieux, Evgenia Passari, Olivier Darmouni, Darrell Duffie for their comments and guidance. He also thanks the participants in internal seminars at Notre Dame, Paris-Dauphine and Collège de France, and participants at the LAGV symposium, ISCFS, the Paris-Dauphine CEPR PhD workshop, MFR Chicago and the HEC Paris PhD workshop for their insights and companionship.

Contents

0 Introduction 3

1 Selva: Production behind an Armed Chokepoint 8

2 Global Regulation and Production 17

3 Blockchain as a Certification Device 31

4 Global Effects of Certification on Prices and Directed Innovation 40

5 Building State Capacity after Certification 48

6 Conclusion 54

Notation Tables 64

A Proofs: Selva: Production behind an Armed Chokepoint 72

B Proofs: Global Regulation and Production 74

C Proofs: Blockchain as a Certification Device 79

D Proofs: Global Effects of Certification on Prices and Directed Innovation 82

E Proofs: Building State Capacity after Certification 85

0 Introduction

A kilogram of gold refined from a licensed mine and a kilogram refined from a pit held by an armed group are the same object. Origin is therefore a claim about a transaction, and nothing in the metal records it. In a market that trades some $361 billion of gold a day (World Gold Council, 2026), a false claim easily finds a buyer. Illegal gold imposes costs that no buyer pays. Across Africa, the commodity boom of 1997–2010 may account for as much as a quarter of the average level of violence (Berman et al., 2017). In the Amazon, mining degrades a forest that may already be near a critical fragmentation threshold (Saravia et al., 2018). Since 2010, U.S., OECD, industry and European rules have each asked the buyers of gold to vouch for its origin (SEC, 2012; OECD, 2016; LBMA, 2012; European Union, 2017). Firms that comply must therefore verify an origin they cannot observe in the metal. The U.S. rule alone was expected to cost issuers $3 to $4 billion at the outset (SEC, 2012). Buyers outside these rules bear none of that cost. The case for these rules is a price argument. If compliant buyers refuse illegal gold, its price falls, and extraction falls with it. Peru illustrates the persistence of this trade: illegally sourced gold was estimated to make up 54 percent of the volume of its gold exports in the first half of 2026 (Samon Ros, 2026). The price argument assumes competitive markets along the chain from buyer to mine. In mining areas controlled by illegal armed groups, miners can instead face restricted access to buyers and essential imports. This paper traces the resilience of illegal extraction from household choices through the intermediary to world prices. It then asks how a physically backed, cryptographically authenticated chain of origin claims changes market access, innovation and the government's incentive to intervene.

I start by modelling such a mining economy; I call the model Selva (Section 1). In it, the militia acts as the de facto government, controlling local trade through coercion. A change in downstream demand need not pass directly to the mine. Miners in Selva grow their own food but pay for essential imports with gold. Where they cannot otherwise afford those imports, their supply of effort bends backward, and a lower price for gold makes them mine more (Proposition 1.3). The group sets the price of gold and the price of imports, chooses which villages to serve and controls coercion. Building on Sánchez de la Sierra (2020), the group chooses these terms to maximize its own profit. Selva replaces the supply curve of the price argument with an implementation problem. Building on interlinked-market and network models (Braverman and Stiglitz, 1982; Kranton and Minehart, 2001; Manea, 2018, 2021), I show how an armed group's control of mineral buyers and essential imports connects household production to downstream policy. Its choices of terms, village coverage and coercion implement regional extraction. The central result of Section 1 is that the group's problem reduces to choosing how much gold to extract, given the least cost of inducing that extraction (Theorem 1.5). A fall in what gold is worth to the group can lower its profit while leaving its output unchanged. Transmission to extraction depends on the relative profitability of feasible quantities. Subsistence and the group's margin are therefore two sources of resilience, and through either one extraction can persist under rules on buyers. The group's selling mechanism sits between the rules and its margin. The rules lower its revenue only through the demand that sets its payments (Theorem 2.10).

Their effect on the price of gold reaches the group's revenue when compliant bids set some of those payments, and stops at the auction when retention and unaffected buyers set all of them.

I first derive the group's optimal selling mechanism (Section 2). Gold is resold to whoever values it most, and the group can disguise the origin of gold it keeps and sell it at a cost. Ausubel and Cramton (1999) show that efficiency is optimal when resale is perfect, and Krishna and Perry (1998) that the Vickrey–Clarke–Groves auction maximizes revenue among efficient mechanisms. I extend both results to a seller who can keep the good. The group's best sale is then a VCG auction offering all complete packages, or equal-sized lots, in which it may retain any package and the residual stock (Theorem 2.3). I model the policy as a compliance cost borne by the final regulated holder, which resale through an unaffected buyer cannot avoid; under this incidence the auction remains optimal after the policy. A winning buyer's payment depends on the alternatives its purchase displaces, including the group's option to keep the gold. When compliant bids fall, the group's total revenue, including the value of retained gold, can fall only through the demand that sets its pivotal payments (Proposition 2.6; Theorem 2.10). A compliant buyer can set the price others pay without buying anything, and what the group earns by laundering enters its own auction as a bid (Proposition 2.7). Retention and outside competition therefore buffer revenue. When retention and unaffected buyers set every payment throughout the policy change, the original production choice remains optimal (Corollary 2.12). In a separate case, revenue and profit can fall while extraction remains unchanged: the original choice still earns more than every feasible alternative, including exit (Corollary 2.13). The section culminates in a global policy floor: the remaining market and laundering can sustain trade even when a policy excludes all regulated buyers. If a surviving outlet remains profitable after full implementation cost and the theorem's uniform cost and value bounds hold, every optimal output has a strictly positive lower bound (Theorem 2.16).

Investigations describe how intermediaries, mixing and recycled-gold classifications can obscure origin after extraction (Federl, 2024; Scheck, 2026). Section 3 asks how credible origin claims change what is sold. Newly mined gold receives a claim at an approved mine; later transfers, processing and recycling inherit its recorded ancestry rather than issuing a fresh clean origin. Under maintained physical identification and custody, this limits downstream relabeling and lets enforcement exclude affected descendants while preserving their history. Inspection and custody support the physical claim; a record alone cannot prevent substitution or establish origin (Proposition 3.1). Without a side payment, a buyer finds reporting conclusive evidence optimal when early reporting avoids a loss or leaves it indifferent; investigation has a return only when early reporting would avoid a loss (Proposition 3.2). A fraud is profitable exactly when its gain exceeds the funded cost of admission and of any silence it buys (Theorem 3.3). In the posted-contract market, when a buyer's avoidable loss exceeds what the coalition that fraudulently admits the gold can save in expected bond forfeiture by buying silence, every finding is reported. Under the stated funding, participation and interiority conditions, the fraud fraction and scrutiny effort have a unique interior equilibrium with partial credibility (Theorem 3.4).

Continuing scrutiny lowers fraud and raises discovery relative to the same contracts without early evidence (Section 3.7). Because scrutiny is a recurring cost, seizures alone cannot finance it, and a cleaner certified stock can cost more in total (Section 3.8). Building on work on records (Halaburda, 2018), costly verification (Townsend, 1979), verifier collusion (Tirole, 1986) and auditor incentives (Duflo et al., 2013), I show how advance finance, collateral and disclosure determine the cost of false admission and who can earn the certified-market premium. These incentives govern the credibility of the admitted claim and the composition of the surviving stock.

Certification breaks the pooling equilibrium and shocks global market conditions for certified and uncertified supply (Section 4). To model the macro effects on the certified and uncertified supplies, I use a North–South model of directed technical change. Before origin can be verified, all gold is sold in one anonymous pool at one price. Once it can be verified, Northern buyers purchase only certified gold, while Southern buyers purchase both. The price of uncertified gold then falls when Northern demand is large enough (Theorem 4.1). This is the difference with rules on buyers (Section 2). Those rules leave the outside price unchanged; certification moves it. When the uncertified price falls, the armed group's receipts and the output of a coerced mine fall with it. A lower price does not always lower the reward to uncertified research, however, because wages can fall too (Section 4.2). Directed technical change predicts that research follows the larger and more profitable market (Acemoglu, 2002). In Acemoglu et al. (2012), clean and dirty inputs are different technologies; here legal and illegal gold are the same metal, and only a credible record of origin separates them. The two sectors also differ in what an inventor keeps. A certified inventor patents a machine design and earns licence fees until a better design replaces it, as in Aghion and Howitt (1992). An improvement at an illegal mine cannot be patented: its owner gains for one period, after which everyone copies it.

Over time, certified technology improves and certified gold becomes abundant. Uncertified gold becomes relatively scarce, so its price and the income of an uncontrolled mine can keep rising, but they grow more slowly than the certified economy (Theorem 4.3). At first, certified research may need public support, and I derive the smallest subsidy that directs research to certified technology (Proposition 4.5). Because each certified invention builds on the previous one, this support can be temporary. Once certified technology is advanced enough, research stays certified without help, and the subsidy can stop after a finite number of periods (Theorem 4.6).

Global effects become local incentives for governments to intervene (Section 5). I model this through the choice that a government with an illegal mine on its territory must make: intervene to retake the region, or leave it to the armed group. Before certification, the government has no incentive to intervene. It has renounced control of the mining region in exchange for protection payments from the armed group. Certification creates an opportunity cost of staying out: a legal mine could operate in the region and pay royalties to the state. I model this legal alternative in three steps. First, a mining company opens the mine only if the royalty leaves it enough profit, which caps the royalty the government can charge (Proposition 5.1). Second, the royalty grows with certified

technology (Corollary 4.4); the government's gain is this royalty net of public services and support for the local households. Third, intervening is costly and may fail. The government therefore compares the protection payment it receives by waiting with the expected value of trying, and of trying again after a failure. This gives a simple threshold: intervention pays when protection income falls below it (Theorem 5.2). The two sides then work like a pair of scissors. On the macro side, protection payments, capped by what the armed group earns from uncertified sales, can grow, but more slowly than the certified economy. On the micro side, the royalty from a legal mine grows with it. Together they push protection payments below the threshold. Every government for which the expected legal value exceeds the cost of an attempt, and which can finance repeated attempts, eventually intervenes. As regions are retaken, the expected area outside state control shrinks geometrically. When it shrinks faster than damage grows at the remaining sites, total damage to protected forest has an explicit bound (Proposition 5.3). Research on commodity prices and conflict studies how prices affect wages, rents and violence (Dube and Vargas, 2013; Berman et al., 2017). Here legal production changes what a government gains by governing, in line with the complementarity of legal and fiscal capacity (Besley and Persson, 2009). I do not model the transition from the certification date to the growth path; the Online Appendix gives a simpler benchmark that compares certification with pooling directly.

A rule that changes who buys gold can redirect trade without reducing extraction. A further route to the pit combines a record bound to the unit at extraction, scrutiny funded well enough to keep that record true, and a legal opportunity that grows faster than protection can pay. Where the state reaches, satellite monitoring with enforcement has reduced deforestation in Brazil (Assunção et al., 2023); the obstacle is reach. Because forest loss is irreversible, control has to arrive while the damage is still within what the forest can absorb.

Gold is only the sharpest case. Where state reach is limited, institutions can alter how a change in demand reaches production. The group's selling arrangements, the evidence behind an origin claim and the government's choice to intervene each shape that transmission. Credible origin claims can also increase the fiscal return to governing. The same questions hang over coltan, timber, cattle and fish, everywhere origin is a story told about a good and is recorded nowhere in it. By making origin credible and legal production more valuable, institutions can change both the market for gold and the fiscal incentive to govern the places where it is mined.

Mining and governance: selected observations in 2023

Different measures; percentages are not national illegal-output rates.





South America

A Colombia: authorization

Unauthorized share of mapped on-land alluvial-gold footprint, 2023.

75.8%

0 100%

79,631 / 105,060 hectares
National monitored footprint.
Area, rather than mineral output.

B Brazil: mining footprint

Garimpos' share of mapped mining area, all minerals, 2023.

64.4%

0 100%

283,791 / 440,416 hectares
National area; MapBiomas Collection 9.
Classification by activity type.

Africa

C Ghana: protected forests

Forest reserves affected by illegal mining, as of May 2023.

11.8%

0 100%

34 / 288 forest reserves
National administrative count.
Affected reserves, rather than the share of forest area destroyed.

D Eastern DRC: armed interference

Estimated workers at sites with armed interference / workers at all visited sites.
Visits: 21 Jan.–20 June 2023.

Gold **87.1%**
24,014 / 27,579 workers; 190 sites

3T **53.4%**
1,896 / 3,553 workers; 36 sites

0 100%

Purposive site sample; 35 of 36 active 3T sites are in Sud-Kivu. Mixed sites excluded.

Figure 1: Selected 2023 observations describe distinct dimensions of mining and governance; the panels do not estimate a common prevalence or trend.

Sources: A, UNODC/MME, pp. 22, 48; B, MapBiomas Collection 9, March 2025 workbook, 2023 column; C, Ghana 2025 VNR, p. 128, May 2023 observation; D, authors' calculations from IPIS on-site records dated 2023.

3T: tantalum, tin and tungsten minerals. DRC coverage differs across minerals and is not nationally representative; the shares do not identify a mineral effect. No sampling-based confidence intervals are implied.

Map boundaries: Natural Earth; equal markers locate countries only. Exact values, definitions, data versions and exclusions are documented in the companion empirical source note.

1 Selva: Production behind an Armed Chokepoint

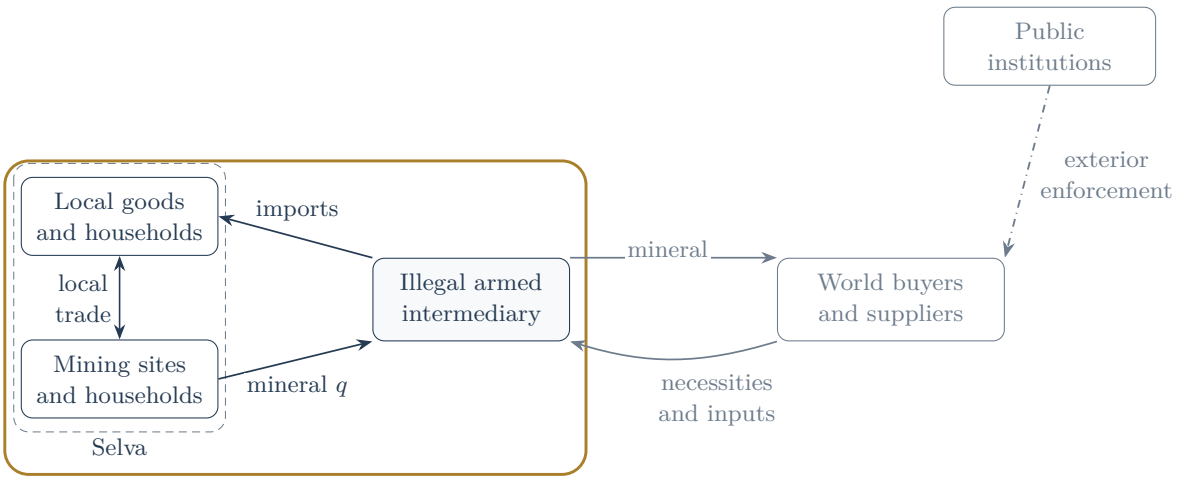


Figure 2: Commodity access and institutional reach: exterior mineral and import routes cross the armed gate, while public enforcement initially reaches exterior actors; gold frames mark the section's focus and muted elements retain context.

1.1 Market Access and Public Authority

Unlawful gold mining can persist or return where extraction is hard to control. Colombia's monitored unauthorized alluvial-gold footprint grew from 58,909 hectares in 2018 to 79,631 in 2023. Visible dredges in a previously mined part of La Pampa rose from 148 to 598 between 2021 and 2023 (Figure 3). The two panels measure different objects, area in one and equipment in the other. They motivate the question of how a mining economy sustains its activity. After the U.S. rule took effect in eastern Congo, militia battles shifted toward gold territories [Parker and Vadheim, 2017].

The model Selva (Spanish for "jungle") describes a region held under the coercive rule of a militia or cartel that sustains itself through illegal mining. The group's monopoly of violence supports a monopoly over the imported share of both mining inputs and the basic consumption basket. Households grow food, exchange local goods and supply their own labor and productive assets, but they buy these essential imports at the group's prices. The group is also the only buyer of the region's mineral. It buys from households at a posted procurement price and controls access to outside buyers, so households cannot sell on the world market. The prices households pay for imports and receive for mineral jointly determine the return to mining [Braverman and Stiglitz, 1982], and the group sets both.

The regional model asks how households' production choices, together with the intermediary that controls their access to trade, determine regional output. Let $\mathcal{G} = (\mathcal{V}, \mathcal{L})$ be the directed network of usable trade routes, with villages and production sites in $\mathcal{J} \subset \mathcal{V}$ and the world market outside it. The OECD's 2021 report on Venezuela describes ELN dominance in the Yapacana mining zone of Amazonas state and reports that the group

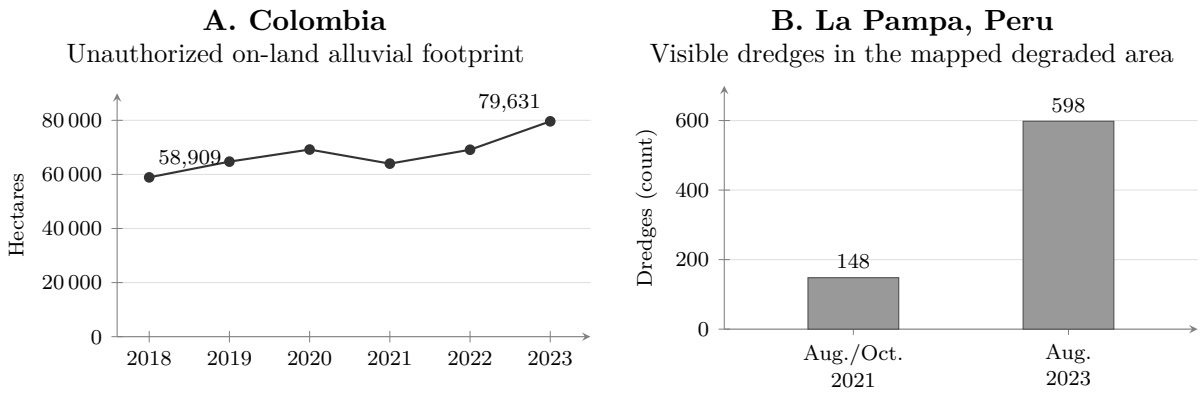


Figure 3: Unauthorized gold activity: expansion and return.

Notes: Panel A reports Colombia's monitored unauthorized on-land alluvial-gold footprint. Panel B counts visible dredges in the mapped degraded La Pampa area. The endpoint increases are 35.2% and 304.1%, respectively. The panels measure area and equipment, rather than tonnes of gold or a causal response to buyer regulation. Renewed activity in previously damaged land can coexist with less new deforestation after Operation Mercury. *Sources:* UNODC/MME, *Explotación de oro de aluvión: Evidencias a partir de percepción remota 2023*, p. 49, Figure 9; Yupanqui et al. (2023), MAAP 193, results and methodology.

largely controlled river shipments of food, fuel and other supplies entering the country [OECD, 2021b, p. 27]. The usable network describes access to trade under territorial control of this kind. A later public intervention can replace the regime that governs access.

Definition 1.1 (A Regional Chokepoint)**.** The region has a chokepoint g if every admissible path carrying mineral from a village in $\mathcal{J}$ to an outside buyer passes through g. The gate is bilateral if every admissible inward path for necessities and mining inputs also passes through g. The armed group controls access to g and can enforce its posted terms there. Villages have no accessible alternative mineral buyer or import supplier on identical delivery and financing terms.

Lemma A.1 shows that, under flow conservation, every external delivery must use gate capacity. Farmers, miners and traders can still exchange goods within Selva, so external monopoly and internal specialization coexist. Available trading paths determine which intermediaries can command rents [Manea, 2018]. In Selva, armed control sustains the external bottleneck.

Existing policies run up against the limits of the state's territorial reach. Even in French Guiana, remoteness and difficult access limit the state's capacity to enforce mining rules [Perrin et al., 2025, p. 42]. Formal buyers, financial intermediaries and importers can face enforceable obligations even where remote extraction sites remain difficult to control. The initial policy experiment isolates this downstream reach.

Assumption 1.2 (Enforcement Reach)**.** During the initial policy experiment, the global policymaker can impose obligations on a specified set of downstream agents. It cannot directly enforce extraction limits, labor rules, procurement contracts or transfers inside the armed-controlled region. Local production technology, the gate's enforceable control and the regional outside opportunities are fixed in that experiment. Establishing public authority inside the region requires a distinct, costly institutional intervention.

1.2 Households, Production and Trading Terms

An exterior policy changes the intermediary's opportunities, and the intermediary then chooses the local terms that serve its profit; later, the same opportunities can change the government's incentive to establish authority inside the region. I first derive the regional response under the existing regime. The gate first chooses which villages to serve from a finite potential set $\mathcal{V}_J$; its active network is $\mathcal{V}^{\text{act}} \subseteq \mathcal{V}_J$. Excluded villages keep their specified outside opportunities and neither supply nor trade with this network. The participating villages form one interconnected economy, in which specialization can make their production complementary, and subsistence must be feasible for every participating household through production and trade across this economy; no single village is required to be self-sufficient. For each attainable mineral quantity q, I find the least cost of inducing its production and compare it with the downstream value of the stock, which determines whether a change in outside demand reaches production. I keep the household and site allocations because output alone describes neither livelihoods nor environmental damage.

At village $v \in \mathcal{V}^{\text{act}}$, household utility is

$$u_v = \log(C_v - c_v) + \eta_v \log r_v + \beta_v \log(b_v + b_{0v}), \qquad C_v > c_v > 0, \quad r_v > 0, \quad b_v \geq 0 \quad (1)$$

Here C_v is consumption and c_v its subsistence threshold. The three terms value consumption above subsistence, leisure r_v, and the sum of newly retained cash b_v and existing savings b_{0v}; the positive weights η_v and β_v govern the value of leisure and of retained cash. The basic consumption basket requires $(1-\alpha)C_v$ units of a local good and αC_v units of imported necessities, where $0 < \alpha < 1$. Households buy the least-cost bundle that produces this Leontief basket, so α measures the imported share of basic needs. Retained cash buys outside consumption at fixed terminal purchasing power. The positive saved balance b_{0v} is set aside for future consumption and cannot finance current spending; spendable opening wealth is $a_v \geq 0$.

The household owns its productive assets. It produces the local good with $F_v(l_v^x)$ and mineral with $G_v(l_v^m, g_v; K_v, A_D)$, subject to

$$l_v^x + l_v^m + r_v \leq 1 \quad (2)$$

Both technologies are continuous and concave. Mineral production combines household labor with inherited equipment K_v and the dirty technological frontier A_D. Running the equipment uses the current consumable input g_v, such as fuel or maintenance.

The intermediary buys mining inputs and imported necessities outside Selva at the given global prices and chooses the prices households pay for them inside the region. It also chooses the price at which it buys mineral inside the region, a common procurement price $\theta_B > 0$. Households take all these prices as given. Each import price equals a village-specific delivered resource cost, which is positive, covers outside purchase and delivery and can differ across villages, plus a common margin chosen by the group. The input margin can be negative while the delivered price stays positive: the gate profits from

such a subsidy because it recovers part of the benefit when it buys the additional mineral, so it chooses procurement and import prices together. The import prices are

$$\begin{aligned} p_v^z &= \underbrace{c_v^z}_{\text{purchase and delivery}} + \underbrace{\mu_z}_{\text{necessity margin}} > 0 \\ p_v^g &= \underbrace{c_v^g}_{\text{purchase and delivery}} + \underbrace{\mu_g}_{\text{input margin}} > 0, \qquad \mu_z \geq 0 \end{aligned} \tag{3}$$

Let p_v be the local-good price and $\bar{p}_v = (1-\alpha)p_v + \alpha p_v^z$ the basket price. Spending on consumption, inputs and retained cash cannot exceed opening wealth plus production revenue. The household solves

$$\max u_v \quad \text{subject to}$$

$$\begin{cases} \underbrace{\bar{p}_v C_v + p_v^g g_v + b_v}_{\text{consumption, inputs and retained cash}} \leq \underbrace{a_v + \theta_B G_v(l_v^m, g_v) + p_v F_v(l_v^x)}_{\text{opening wealth and production value}} \\ l_v^x + l_v^m + r_v \leq 1 \\ C_v > c_v, \quad r_v > 0, \quad l_v^x, l_v^m, g_v, b_v \geq 0 \\ \text{technological feasibility} \end{cases} \tag{4}$$

Each source of income affects consumption and, in turn, the mineral the household extracts. Rights to innovation income are specified separately from ownership of existing equipment.

Proposition A.2 characterizes these choices. The household allocates its time and inputs to maximize utility, so a change in θ_B acts on supply only together with the import prices, local prices and participation conditions that determine what mining income buys.

1.3 Subsistence and Mining Effort

A household that depends on mining to meet its subsistence needs may have to work more when the purchasing power of its mining income falls. Basu and Van [1998] emphasize this survival motive; here I apply it to household mining effort. To isolate it, I abstract from local trade, productive inputs and the utility of cash and vary the mining return ceteris paribus.

Let mining be linear, with earning power $w = \theta_B A_m$, and let producing the local complement of one basket take $d^x = (1-\alpha)/A_x$ units of time. Write $B^{\text{sub}} = \alpha p^z$ for the price of the imported component. With wealth a, positive mining labor l finances $B^{\text{sub}} C = a + wl$, and leisure is $r = 1 - d^x C - l$. On a strictly feasible interior branch, I define $R^{\text{time}} = 1 - d^x a / B^{\text{sub}}$ and $k_1 = 1 + d^x w / B^{\text{sub}}$.

In this interior benchmark, a fall in the mining return raises effort and output when wealth does not exceed the imported-subsistence bill, holding the other parameters fixed.

Proposition 1.3 (The Subsistence Labor Response)**.** *In this benchmark the interior optimum satisfies*

$$\begin{aligned} \underbrace{l}_{\text{mining labor}} &= \frac{R^{\text{time}}}{(1+\eta)k_1} + \frac{\eta(B^{\text{sub}}c - a)}{(1+\eta)w} \\ \underbrace{l_w}_{\text{response to earning power}} &= -\frac{R^{\text{time}}d^x}{(1+\eta)B^{\text{sub}}k_1^2} - \frac{\eta(B^{\text{sub}}c - a)}{(1+\eta)w^2} \end{aligned} \tag{5}$$

In particular, if $a \leq B^{\text{sub}}c$*, a higher mining return reduces required mining labor on that branch. More generally, the response is positive exactly when* $a - B^{\text{sub}}c > R^{\text{time}}d^x w^2/(\eta B^{\text{sub}}k_1^2)$*, negative when the inequality is reversed, and zero at equality (see Appendix A.1.1).*

Once wealth covers subsistence, the substitution effect can dominate, and a lower return can then reduce effort. Both responses are confined to a strictly feasible interior allocation; the subsection "The no-mining corner of the distress benchmark" in Online Appendix A.1 treats the boundary at which mining ceases.

In Selva, however, the group chooses the procurement price and the import prices jointly. Its monopsony over mineral and monopoly over imports let it adjust these terms when downstream opportunities change. Household effort responds to the resulting terms, subject to subsistence, participation and production constraints. I next derive the cost of the feasible allocations among which the armed group chooses to maximize profit; Section 2 establishes when its preferred output survives a downstream demand restriction.

1.4 Village Trade and Participation

In the linked economy, households can specialize and trade, local prices adjust, and the gate must induce participation in the villages it serves. These adjustments determine which regional allocations can be sustained. Local goods move on a specified finite directed village network. Shipping t_{vw} units from v delivers t_{vw}/δ_{vw} to w, with $\delta_{vw} \geq 1$. A competitive linked equilibrium combines household optima with market clearing and zero-profit shipment conditions:

$$\begin{aligned} &\underbrace{F_v}_{\text{local production}} + \underbrace{\sum_w t_{wv}/\delta_{wv}}_{\text{delivered inflows}} - \underbrace{\sum_w t_{vw}}_{\text{outgoing shipments}} = \underbrace{(1-\alpha)C_v}_{\text{local-good consumption}}, \\ &\underbrace{p_v - p_w/\delta_{vw} \geq 0}_{\text{no profitable shipment}}, \qquad \underbrace{t_{vw}(p_v - p_w/\delta_{vw}) = 0}_{\text{trade only at zero profit}} \end{aligned} \tag{6}$$

The prospect of shipment limits price differences even on routes that carry no trade. Free disposal can be included with zero-value complementarity. When transport is costless,

removing inessential shipment cycles preserves household allocations and market-clearing values.

Trade can make survival possible where isolated villages would fail. Without outside wealth, a farming village needs mineral earnings to pay for imported necessities, and a mining village without local production needs the farmer's good. Their exchange lets the miner's export earnings finance both households' requirements. Online Appendix A.1 constructs a smooth two-village economy with a unique market-clearing local price. The viable economic unit is then the village network.

Household v has outside utility $\bar{U}_v$. The gate can apply coercive effort e_v at real cost $\Psi_v(e_v)$, so participation requires

$$u_v \geq \bar{U}_v - \Delta_v(e_v), \qquad 0 \leq e_v \leq \bar{e}_v \tag{7}$$

The function Δ_v, continuous and strictly increasing with $\Delta_v(0) = 0$, measures how far coercion worsens the outside option. Its cost Ψ_v is continuous and increasing, with $\Psi_v(0) = 0$. Coercion can make refusal less attractive, but induced production must still satisfy subsistence, the time constraint and the household's demand for positive leisure.

Proposition A.3 gives the least coercive effort that supports an allocation and the limit imposed by the effort cap. A larger trading margin can raise this cost, because worse household terms require more coercion when participation binds, and the cap can make continued participation infeasible. The illegal group therefore compares village networks by total profit, including the trading terms and the coercion needed to keep their households active.

Equation (3) records the total response to a change in common terms on a regular equilibrium branch. When a village enters or a binding constraint changes, the gate compares the feasible allocations directly.

1.5 Resources and Implementation Cost

Summing over participating villages gives total mineral output, imported necessities and purchased mining inputs, respectively:

$$q = \underbrace{\sum_{v \in \mathcal{V}^{\text{act}}} G_v}_{\text{mineral output}}, \qquad Z = \underbrace{\sum_{v \in \mathcal{V}^{\text{act}}} \alpha C_v}_{\text{necessity imports}}, \qquad I = \underbrace{\sum_{v \in \mathcal{V}^{\text{act}}} g_v}_{\text{imported mining inputs}} \tag{8}$$

The gate pays $\theta_B q$ for mineral, earns the necessity and input margins, and pays for mineral transport, coverage and coercion. Let z collect its terms, active set, coercion and the

induced competitive equilibrium, and let $T(z) \geq 0$ be its transport and coverage bill. The cost of implementing this allocation is

$$C^{\text{impl}}(z) = \underbrace{\theta_B q}_{\text{mineral procurement payments}} - \underbrace{\mu_z Z}_{\text{necessity margin receipts}} - \underbrace{\mu_g I}_{\text{net receipts on inputs}} + \underbrace{T(z)}_{\text{mineral transport and coverage}} + \underbrace{\sum_v \Psi_v(e_v)}_{\text{coercion costs}} \tag{9}$$

The armed group earns on both sides of the household budget: it pays θ_B for mineral and collects margins on the imports that mining income buys. Because mineral sales, imports and input purchases all respond to the prices it sets, part of every procurement payment comes back as import margin, and its net cost is a nonlinear function of those prices. Consolidating the regional budgets shows which expenditures remain once internal transfers cancel.

Proposition 1.4 (Consolidated Resources)**.** *Every linked equilibrium satisfies*

$$\theta_B q + \sum_v a_v = \sum_v (p_v^z \alpha C_v + p_v^g g_v + b_v) \tag{10}$$

$$C^{\text{impl}}(z) = \sum_v (c_v^z \alpha C_v + c_v^g g_v + b_v - a_v) + T(z) + \sum_v \Psi_v(e_v) \tag{11}$$

The identities hold with iceberg losses on local routes (see Appendix A.1.2).

In (10), mineral earnings and opening wealth pay for imports at posted prices and for the cash households retain. In (11), the import margins cancel, and the gate's net cost reduces to real resource costs, net of households' opening wealth. The gate can therefore profit either through high monopoly prices on imports or through low monopsony procurement prices; a high nominal mineral payment returns to it through tied imports. Household cash enters the illegal group's private cost alongside real resource spending, so the cost is not a measure of regional welfare (which is not among the armed group's objectives).

If $W(q)$ is the gross value of delivering stock q to the downstream stage, the group's objective is $\Pi(z) = W(q(z)) - C^{\text{impl}}(z)$. The gate chooses among outputs whose local allocations it can implement. Let $\mathcal{Z}$ collect the competitive equilibria that its prices and other trading conditions can sustain. Where an institution selects among multiple equilibria, I fix its selection rule independently of the downstream policy, so the opportunities for regional implementation stay constant across the policy comparison.

For output q with at least one admissible implementation, define

$$\mathcal{C}(q) = \min_{z \in \mathcal{Z}: q(z) = q} C^{\text{impl}}(z), \qquad \Gamma^{\text{impl}}(q) = \arg\min_{z \in \mathcal{Z}: q(z) = q} C^{\text{impl}}(z) \tag{12}$$

Here C_v denotes household consumption and $C^{\text{impl}}(z)$ the gate's net cost of one allocation. In (12), $\mathcal{C}(q)$ is the least cost over all admissible implementations of the given output, and $\Gamma^{\text{impl}}(q)$ collects every implementation that attains it. Acting as a de facto social planner

with a private profit objective, the armed group chooses the regional output level and its cost-minimizing allocation, subject to the fixed equilibrium-selection rule where one applies.

The comparison needs the minimum to be attained. Assumption A.4 and Theorem A.5 in Online Appendix A.1 give sufficient conditions for a smooth linked class. Subsistence, positive resource costs and diminishing returns bound the relevant choices, and the closed equilibrium graph preserves feasibility at their limits. Under those conditions every nonempty positive-output implementation set attains its minimum. With the theorem's bound on downstream value and its upper semicontinuity condition, a regional profit maximum also exists, which allows finite coverage comparisons and the separate exit option.

The subsistence-financing floor is the output below which the region cannot finance its imported-subsistence deficit in (5) without operating at a loss, given the theorem's upper bound on downstream value. The deficit arises when outside wealth falls short of the imported-subsistence bill, and farmers and miners may finance it jointly through trade. Attainment on a nonempty implementation graph is a separate matter from showing that the graph is nonempty. For a smooth subclass, a linked-price construction supplies a continuous selected equilibrium (see Online Appendix A.1).

An allocation can be supported by more than one gross procurement quote and import price. In the nominal-price example in Online Appendix A.1, different price pairs sustain the same household choices and resource cost. The cost comparison therefore keeps a set of implementing terms, and uniqueness of a particular quote requires a separate argument.

1.6 Regional Output and Physical Allocations

After procurement, the gate values the stock through the trading institutions available to it as a seller. The seller offers every complete package of size $\bar{q}$, so offered capacity is $\lfloor q/\bar{q} \rfloor$, as defined in Equation (15) of Online Appendix A.2. Under efficient resale this restriction is without loss of generality, because committed withholding cannot raise the seller's continuation value (Online Appendix, Proposition A.16(1)). The reduction below keeps the attainment and existence conditions of Assumption A.4 and Theorem A.5 and adds attainment of the downstream capacity maximum.

In (13), the first choice selects an operating quantity that maximizes downstream stock value less its least implementation cost, with exit compared at its separately specified payoff. The second selects a cost-minimizing local implementation of the chosen quantity.

Theorem 1.5 (Regional Output as an Implemented Choice)**.** *Suppose the output fibers and downstream capacity maxima attain and the regional maximum exists under the conditions above. The full joint choice of posted terms, village coverage, coercion, equilibrium allocation and offered capacity is equivalent to*

$$q^* \in \arg\max_{q \in \mathcal{Q} \cup \{0\}} \{W(q) - \mathcal{C}(q)\}, \qquad z^* \in \Gamma^{\text{impl}}(q^*) \tag{13}$$

with the separately specified exit payoff at $q = 0$. An unchanged optimal quantity leaves the implementation correspondence $\Gamma^{\text{impl}}(q^*)$ unchanged in a pure downstream experiment. It implies unique unchanged posted terms only when those components of $\Gamma^{\text{impl}}(q^*)$ are single-valued (see Appendix A.1.3).

Each candidate quantity carries its least cost and a feasible pattern of livelihoods and control, so output is an implemented choice. With regional primitives fixed, an unchanged optimal output leaves the set of cost-minimizing implementations unchanged. The selected household allocation, the trading terms and the environmental damage stay fixed only if the implementation is unique or the selection from that set is unchanged.

The physical allocation matters because mineral output, mining labor, purchased inputs and operating sites measure different margins of damage. Saving a harmful input can improve environmental outcomes at unchanged output, while opening a site can increase damage without changing measured productivity. The conservation criterion therefore applies to the reconstructed household and site allocation, and it is defined by physical activity at the protected sites.

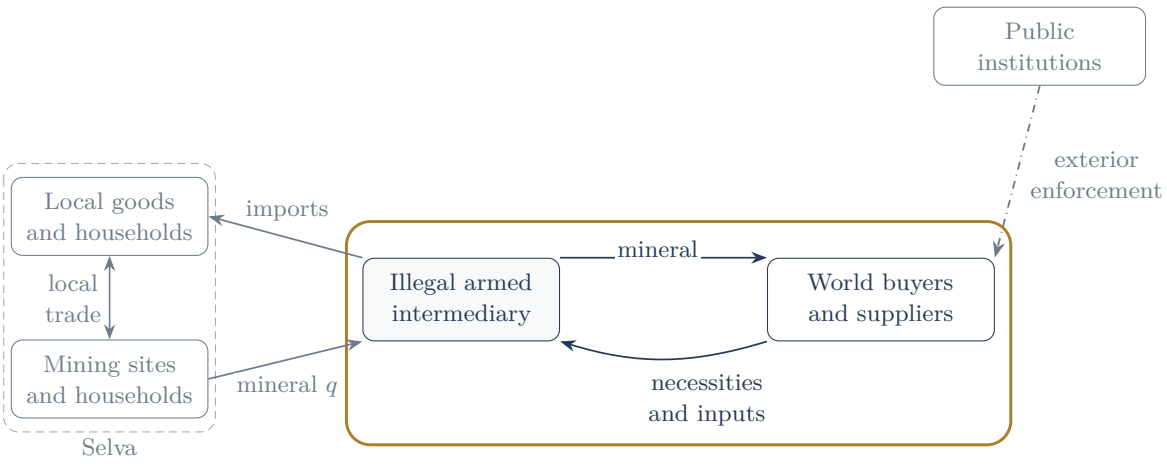
Assumption 1.6 (A Specified Physical Conservation Target)**.** Let $\mathcal{E}$ be the designated set of ecologically critical Selva sites. Each site has a stated safe physical allocation set $\mathcal{A}_r^{\text{safe}}$, describing extraction, land use, inputs and livelihood requirements. Conservation means that the reconstructed allocation belongs to this set at every designated site under the asserted date or limiting criterion. Fiscal gains, certified labels and a lower global contamination fraction are not themselves this condition.

Section M returns to this criterion with funded household expenditures and specified safe production locations. Proposition M.2 gives finite dates for reaching each positive tolerance of aggregate protected-site disturbance, and a separate condition on cumulative damage. With imperfect intervention, some ungoverned sites can remain at every finite date.

The regional cost and implementation correspondence also organize the later world-market analysis, whose closed-form supply uses a separate specialization. Proposition A.8 derives that envelope from installed equipment with no variable household mining labor, necessity prices at resource cost, no productive-input subsidy and nonmining resources sufficient for baseline survival. The subsistence benchmark isolates an effort response, and the smooth linked class supplies sufficient conditions for attainment; neither alone supplies the homogeneous cost used in Section L.

A restriction on buyers lowers the illegal group's downstream return $W(q)$ only when regulated bids set its payments; payments can stay unchanged while unaffected demand or retention determines them. Section 2 asks how competition among unaffected buyers and the group's opportunities to retain and launder mineral shape $W(q)$, and when the resulting change moves the gate's chosen quantity relative to all attainable alternatives.

2 Global Regulation and Production



Retention and competing buyers shape how selective regulation reaches the gate.

2.1 The Production Question and the Policy Experiment

A downstream restriction lowers regulated buyers' willingness to pay, but does not mechanically change the armed group's regional procurement price. Section 1 examined how a lower return to mining affects household effort and how the gate sets the regional prices and trading conditions that support production. At the selling stage the procured gold is a fixed stock for which several independent buyers may compete, so a fall in some buyers' willingness to pay can send the same gold to others. I ask how far the restriction reaches the return to extraction, studying this competition first and then the gate's choice of how much to obtain from Selva.

The spatial price spread $0 < \theta_B < \theta_E$ creates the initial trading opportunity. The armed group buys mineral inside the region at its posted procurement price θ_B, which households take as given and the gate chooses together with its other regional terms. The outside quote θ_E is the world-market price of physical gold outside the gate, per standardized physical unit. It is a gross resale benchmark, because the costs of making the gold eligible for that market and of gaining access to it remain to be paid. The regional lot is small relative to the market. The outside quote is fixed within a trading round; world prices adjust across dates and policies.

The gold market trades some $361 billion a day [World Gold Council, 2026], which motivates this assumption. LBMA describes trading between market institutions once eligible Good Delivery bars enter an approved London vault [London Bullion Market Association, 2026]. Reaching that market still requires refining, delivery, finance, compliance and access, and the model counts their costs (see Online Appendix G.1).

Competition can limit how far a selective demand shock passes to the illegal group's receipts. A winning buyer pays for the alternatives its allocation displaces, including the

demand of buyers who receive no gold. When regulated orders set those payments, a compliance charge can lower them; when unaffected orders take their place, declines can be limited or stopped. I call this limited transmission to seller value *resilience*.

Gold also keeps a market value outside the auction, because the gate can conceal its origin and sell it into the general economy at a cost. With storage, this self-laundering opportunity gives retained gold a value that can put a further floor under payments. Under conditions given below, these outlets hold payments above acquisition cost, and they preserve extraction when the chosen quantity stays at least as profitable as every feasible alternative, exit included. The gate's profit can fall while that comparison still holds, and a downstream demand restriction then produces no corresponding contraction at the mine.

A fixed set of regulated buyers $\mathcal{R}$ faces compliance cost $\tau^{\text{pol}} \geq 0$ per standardized unit of which it is the final holder. Buying through an unaffected intermediary does not avoid this charge. Buyers in $\mathcal{U}$ and laundering are unaffected. The illegal group launders through regions the policy does not reach. Because the mineral is indistinguishable without a reliable record of origin and regulators have no direct access to the mines, this policy describes current frameworks.

2.2 Buyers, Retained Stock and the Selling Institution

The gate holds physical stock q and sells packages of size $\bar{q}$. The number of complete packages and the remainder too small to form another package are

$$Q = \lfloor q/\bar{q} \rfloor, \qquad r_q = q - \bar{q}Q, \qquad 0 \leq r_q < \bar{q} \tag{14}$$

The quotes θ_B and θ_E are prices per standardized physical unit, marginal buyer values are amounts per package, and transfers are total payments. Procurement expenditure $\theta_B q$ is paid before bidding.

Buyers differ in processing costs, financing needs and resale opportunities. Before bidding, each buyer privately knows its operating conditions; neither the seller nor rival buyers observe them. The finitely many distinct owners form $\mathcal{I}$, with $N = |\mathcal{I}|$. Buyer i has public capacity $\bar{k}_i$, the number of packages it can take, and private type ω_i, which summarizes its operating opportunities. Its operating value $J_i^0(x; \omega_i)$ is net of the specified processing and financing costs, includes resale, and is measured before payment for the auctioned mineral. A marginal package value is the increase in that operating value, and a buyer's value adds the values of the packages acquired and deducts the compliance bill when the buyer is regulated:

$$\begin{aligned} b_i^k(\omega_i) &= J_i^0(k\bar{q}; \omega_i) - J_i^0((k-1)\bar{q}; \omega_i), \\ v_i(k; \omega, \tau^{\text{pol}}) &= \sum_{j=1}^{k} b_i^j - \underbrace{\bar{q}\tau^{\text{pol}} k \mathbf{1}_{\{i \in \mathcal{R}\}}}_{\text{compliance bill}} \end{aligned} \tag{15}$$

Concavity of J_i^0 makes these increments decrease, and a buyer whose post-policy value turns negative can receive zero units. A buyer with access to an unconstrained outside source of identical mineral at θ_E has an opportunity-adjusted marginal package value of at most $\bar{q}\theta_E$, so the value of access inside the auction depends on access outside it.

All inventories and policy levels share the same buyers, capacities and type law ν_ω, which describes the common belief about other buyers' private operating opportunities at the time of bidding. Realized valuations stay private (see Online Appendix G.2). In the committed VCG mechanism below, truthful reporting of one's own values is weakly dominant. I also assume the expected sum of positive marginal package values to be finite:

$$\mathbb{E}_{\nu_\omega}\left[\sum_{i\in\mathcal{I}}\sum_{k=1}^{\bar{k}_i}(b_i^k)_+\right] < \infty$$

With finite retention value, this condition keeps expected receipts and their policy changes finite (see Online Appendix G.3).

The relevant alternative to an auction sale is the best use of retained stock, which laundering, storage and disposal together use up. I write its gross value as

$$H(y) = \max_{\ell+s+d^{\text{disp}}=y}\{\theta_E\ell - V(\ell) + S^{\text{stor}}(s)\}, \qquad \ell, s, d^{\text{disp}} \geq 0 \tag{16}$$

Laundering quantity ℓ incurs real cost $V(\ell)$, storage s has net continuation value $S^{\text{stor}}(s)$, disposal d^{disp} is free, and channel capacities constrain the same allocation problem. "Gross" here means before regional implementation cost (see Online Appendix G.4). I assume that H is finite, continuous, concave and nondecreasing on the relevant domain; convex laundering costs, concave storage returns, convex capacities and free disposal give these properties when the optimum is attained.

Unsold packages and the residual stock r_q compete for the same channels. With procurement sunk, a package's reservation value is therefore its incremental gross retention return $H(y+\bar{q}) - H(y)$ at the stock already retained. When laundering is the only retention use chosen at both stock levels, this return is the maximum revenue from laundering the package, $\bar{q}\theta_E - [V(y+\bar{q}) - V(y)]$, which exceeds the package procurement bill if and only if incremental laundering cost is below $\bar{q}(\theta_E - \theta_B)$ (Lemma A.9). The spatial spread creates a potential market outlet, and channel costs determine how much of the spread the seller preserves.

Before buyers report, the seller commits to offer all Q complete packages, use its genuine retention schedule H, and allocate efficiently with Clarke pivot payments. Buyer i pays the welfare loss its allocation imposes on the rest of the system, where $S_{n,-i}$ is the maximum

total value achievable without buyer i. The seller's receipts add the value of the stock it retains, and their expectation is taken before buyers' private types are known:

$$S_n(q,\omega,\tau^{\text{pol}}) = \max_{\substack{0\le k_i\le \bar{k}_i,\ k_i\in\mathbb{Z} \\ \sum_i k_i\le n}} \left\{ \sum_i v_i(k_i;\omega,\tau^{\text{pol}}) + H(q-\bar{q}\textstyle\sum_i k_i) \right\} \tag{17}$$

$$p_i = S_{n,-i} - S_n + v_i(k_i^*;\omega,\tau^{\text{pol}}),$$

$$R_n(q,\omega,\tau^{\text{pol}}) = \underbrace{\sum_i p_i}_{\text{buyer payments}} + \underbrace{H(q-\bar{q}\textstyle\sum_i k_i^*)}_{\text{actual retained value}},$$

$$W_n(q,\tau^{\text{pol}}) = \mathbb{E}_{\nu_\omega} R_n(q,\omega,\tau^{\text{pol}}) \tag{18}$$

The stock-value function used in Section 1 offers every complete package at each physical stock (details in Online Appendix A.2).

Proposition 2.1 (Incentives within the Committed Mechanism)**.** *Truthful reporting is weakly dominant for every buyer, and a truthful buyer has nonnegative utility. These properties hold after the announced common compliance cost. They do not imply that the mechanism is the unique revenue-maximizing institution available to the seller (see Appendix B.1.1).*

I use the complete-game approach of Ausubel and Cramton [1999] and the efficient-payment comparison of Krishna and Perry [1998] to show that the mechanism is optimal under my assumptions. I compare procedures whose complete outcomes allocate stock efficiently, with public nonlinear retention value H and the whole-buyer transfers in (18). The comparison first sets aside the selective compliance charge.

Assumption 2.2 (Fixed-Stock Efficient-Resale Benchmark)**.** Initially $\tau^{\text{pol}} = 0$ and q is already procured. The following restrictions apply to the institution comparison.

1. True H, stock, capacities and the buyer laws are public before mechanism choice. Buyer marginal vectors are independent private types on compact convex domains of nonnegative decreasing values containing zero. All types, including zero, can abstain from every trade at zero utility.

2. The seller commits to initial terms and cash settlement. All Q packages, including initially unoffered or unsold packages, remain available for voluntary terminal trade before any irreversible use. Final feasible quantities and channel opportunities are the same across procedures.

3. Private transfers balance; preferences are quasilinear; all costs and outside operating opportunities are counted once. Initial nonparticipants can join resale. There is no new outside signal revealing true types after auction reports.

4. The comparison is over complete-game Bayesian Nash equilibria whose terminal allocations are efficient relative to true values and H, as in Definition A.10. This is

a substantive perfect-resale outcome restriction, not a claim that liquidity or a finite bargaining protocol guarantees efficiency after every selling procedure.

Theorem 2.3 (Optimality of Genuine-Retention VCG under Efficient Resale)**.** *Under Assumption 2.2, every admissible complete outcome gives the seller expected total continuation payoff at most $W_Q(q,0)$. The all-offered VCG auction with genuine H and the whole-buyer pivot payments in (18) attains this bound. Specifically, when terminal trade requires final voluntary consent to a balanced contract against initial ownership, truthful buyer reports and no trade form an ex post Nash equilibrium of the fixed-procedure auction-and-resale game. The equilibrium resists combined misreport/resale and nonparticipation/resale deviations. No additional strategic reserve premium or participation charge is needed on this domain.*

To compare a genuine retention announcement with an alternative, I count the seller's net cash transfer T_0^{net} over the auction and later trade, evaluated at the induced complete equilibrium, plus its final retained stock valued at true H:

$$\Pi_q(\widehat{H};H) = \mathbb{E}\left[T_0^{\text{net}}(\widehat{H}) + H(y^{\text{final}}(\widehat{H}))\right] \tag{19}$$

Corollary 2.4 (An Optimal Genuine Announcement)**.** *Under Assumption 2.2, select the attaining equilibrium at the genuine announcement. For every alternative announcement and each of its admissible complete equilibria,*

$$\Pi_q(H;H) = W_Q(q,0) \geq \Pi_q(\widehat{H};H) \tag{20}$$

Thus declaring the joint retention increments without an additional strategic premium is an expected-payoff optimal choice. Neither uniqueness nor dominant-strategy truthfulness of the revenue-receiving seller is asserted.

The bound comes from the participation rent of the lowest type [Krishna and Perry, 1998, Ausubel and Cramton, 1999]. On the stated domain, every efficient incentive-compatible procedure gives each buyer an interim expected utility equal to its interim pivot utility plus a nonnegative participation constant. With balanced cash transfers, any such constant comes out of the seller's payoff, and VCG sets the constants to zero (see Appendix B.1.2; full game details are in Online Appendix A.2.2 and G.5). The genuine-retention VCG auction uses no additional reserve premium and attains the highest seller payoff within this efficient-resale class.

An independently disclosed H fixes the seller's opportunity cost before the mechanism is chosen, whereas a privately informed seller would need an information rent of its own, and balanced transfers cannot in general finance both parties' truthful participation rents while preserving efficiency (Online Appendix, Proposition A.15) (see Online Appendix G.6).

Throughout, τ^{pol} attaches once to the final regulated holder and cannot be avoided by buying through an unaffected intermediary, as under due-diligence rules that bind the importer or user of record. With this incidence, Theorem 2.3 extends to every $\tau^{\text{pol}} \geq 0$

with the unchanged pivot rule (Online Appendix, Proposition A.16(3)), and the VCG auction remains the seller's best institution after the policy (see Online Appendix G.7). I now analyze resilience: the capacity of this mechanism to limit the loss in payments and production that a regulatory shock causes.

The robustness of the resilience results does not depend on the VCG auction. In a companion paper I prove, with the optimal mechanism of Loertscher and Muir [2022], that the same resilience results for profit and production survive even under limited resale [Lucio, 2026]. Although the VCG analysis rests on stronger resale assumptions, truthful reporting remains weakly dominant for each buyer after the policy shock, which lets me identify the demand that determines payments. The results assume individual incentive compatibility; coordinated bidding by buyer rings would require separate controls.

2.3 Who Receives the Gold, and Who Determines Its Payment?

For retained stock $y \geq 0$, the seller values its j^{th} additional package by its marginal contribution to retention value:

$$\rho_j(y) = H(y + j\bar{q}) - H(y + (j-1)\bar{q}), \qquad j = 1, 2, \ldots \tag{21}$$

Let $y_0 = q - n\bar{q}$ be the stock outside the offered capacity, fractional residual included. The n optional seller orders are $\rho_1(y_0), \ldots, \rho_n(y_0)$, an empty sequence when $n = 0$. For $n \geq 1$, the last and smallest seller order is the one-package reserve $\rho(q) := H(q) - H(q - \bar{q}) = \rho_n(y_0) = \rho_1(q - \bar{q})$. Concavity of retention makes these orders nonnegative and decreasing. The n packages go to the n highest orders, the seller's own orders included, with a fixed tie-breaking rule.

Lemma 2.5 (Whole-Buyer Critical Payments)**.** *For buyer i receiving $k_i^* > 0$ packages, let $z_{-i,[j]}$ be the j^{th} highest order after removing all of i's orders, including its losing ones. Index these packages by $u = 1, \ldots, k_i^*$. Its transfer is*

$$p_i = \sum_{u=1}^{k_i^*} c_{iu}, \qquad c_{iu} = z_{-i,[n-k_i^*+u]} \tag{22}$$

The terms in this sum are the additional rival orders filled when buyer i is removed (see Appendix B.1.1).

The seller's competitive alternatives depend on who owns the orders. For resilience, the relevant demand is what remains when an owner is removed.

Proposition 2.6 (Reallocation Toward Unaffected Buyers)**.** *At fixed q, n, H and realized types, an unaffected buyer's allocation weakly increases when τ^{pol} increases, with the same fixed priority at ties. Under any fixed type law its post-policy allocation therefore first-order stochastically dominates its pre-policy allocation (see Appendix B.2.1).*

Unaffected buyers keep at least the same allocation. Whether the gate keeps obtaining the same amount of gold is the profit comparison developed below.

2.4 How Alternative Outlets Support Payments

Let $m^{\text{sold}} = \sum_i k_i^*$ count the packages sold to buyers and $y = q - \bar{q}m^{\text{sold}}$ the physical stock retained. Taking back one winning buyer's entire block would raise retention value by

$$D_i(y) = H(y + k_i^*\bar{q}) - H(y) = \sum_{u=1}^{k_i^*} \rho_u(y) \tag{23}$$

The global price sets the maximum gross return from laundering; the cost of using the channel determines the margin. When laundering is the only interior retention choice, the u^{th} package's incremental retention value per physical unit is θ_E net of the incremental laundering cost per unit:

$$\frac{\rho_u(y)}{\bar{q}} = \theta_E - \frac{V(y + u\bar{q}) - V(y + (u-1)\bar{q})}{\bar{q}}$$

Proposition 2.7 (The Opportunity Floor)**.** *Every winning buyer satisfies*

$$c_{iu} \geq \rho_u(y), \qquad p_i \geq D_i(y), \qquad R_n(q, \omega, \tau^{\text{pol}}) \geq H(q) \tag{24}$$

Thus a positive retention opportunity margin above acquisition cost supports a positive acquisition margin on the externally sold block. A strictly positive competition premium over retention requires strictly stronger rival orders; a winner's own willingness to pay above the reserve is not sufficient (see Appendix B.2.2).

The acquisition margin per unit splits into a part supported by retention and the competition premium:

$$\frac{p_i}{k_i^*\bar{q}} - \theta_B = \underbrace{\frac{D_i(y)}{k_i^*\bar{q}} - \theta_B}_{\text{retention acquisition margin}} + \underbrace{\frac{p_i - D_i(y)}{k_i^*\bar{q}}}_{\text{competition premium}} \tag{25}$$

Retention can sustain a positive acquisition margin when the competition premium, which is weakly nonnegative, is zero. Whether that margin supports production depends on the full regional cost.

Each buyer pays at least the value of retaining its block, with other sales held fixed. Concavity allows these separate block values, all evaluated from the same retained stock, to exceed the value of retaining the sold gold together. The aggregate premium $R_n - H(q)$ can therefore be positive even when every buyer pays only its block's retention value (see Online Appendix, equation (24)).

Let $\ell(y)$ be the laundering quantity selected in (16). I use $p^R = \theta_E - \tau^{\text{pol}}$ as a scalar index; it is not a common auction payment. Actual laundering is the selected laundering use at the stock left after buyer sales:

$$L(q, p^R, \omega) = \ell(q - \bar{q} m^{\text{sold}}(q, p^R, \omega)) \tag{26}$$

At fixed stock, laundering changes only in discrete steps and stays constant as long as the integer allocation is unchanged, and a binding laundering capacity also limits its response. Buyers keep competing for mineral in either case. The first derivative below holds physical stock fixed, and the second adds the effect of any change in the illegal group's chosen production.

Proposition 2.8 (A Zero Direct Laundering Response)**.** *On a differentiable regime with fixed offered capacity,*

$$L_{p^R}\Big|_q = -\bar{q}\ell'(y) m^{\text{sold}}{}_{p^R}\Big|_q, \qquad \frac{dL(q^*(p^R), p^R)}{dp^R} = L_{p^R} + L_q \frac{dq^*}{dp^R} \tag{27}$$

Thus a zero direct laundering response is consistent with changing payments, positive profits and changing regional production. Under uncertainty, statewise constancy between allocation switches does not generally imply constancy of expected laundering (see Appendix B.2.1).

To bound the payments supported by unaffected demand, I remove each buyer in turn and, for $n \geq 1$, take the lowest of the n^{th} highest remaining unaffected orders across these exclusions:

$$e_{\mathcal{U}}(n, \omega) = \min_i b_{\mathcal{U}\setminus\{i\},[n]} \tag{28}$$

The subscript selects the n^{th} highest pooled marginal order, set to zero if fewer than n orders exist. Adding the seller's n orders to each unaffected rival book gives the minimum $e_{\mathcal{U},H}(q, n, \omega)$. At fixed inventory and retention technology, the regulated-buyer cost moves neither floor.

Proposition 2.9 (The Remaining-Market Floor)**.** *For every externally sold package and every $\tau^{\text{pol}} \geq 0$,*

$$c_{iu} \geq \max\{\rho_u(y), e_{\mathcal{U},H}(q, n, \omega)\}, \qquad e_{\mathcal{U},H} \geq e_{\mathcal{U}} \tag{29}$$

In particular, if $e_{\mathcal{U},H} > \bar{q}\theta_B$, every externally sold package has a strictly positive acquisition margin, including a regime on which the direct laundering response is zero (see Appendix B.2.2).

Competition outside the regulated group can therefore hold payments up after the laundering response has stopped. For packages sold with certainty, an expected floor above acquisition cost supports a positive expected placement margin (see Online Appendix G.8). If n buyer orders also exceed the highest seller order in every state, the whole offered inventory sells externally with certainty.

2.5 How Regulation Reaches the Seller's Value

The buyers who receive mineral need not be the buyers whose demand sets payments. Regulation reaches payments when it lowers the value of orders that would replace an excluded buyer. Let k^{-i} be the allocation that excludes buyer i, with $k_i^{-i} = 0$, and write $m^{\text{sold}}{}_{\mathcal{R}} = \sum_{j\in\mathcal{R}} k_j^*$ and $m^{\text{sold}}{}_{\mathcal{R}}^{-i} = \sum_{j\in\mathcal{R}} k_j^{-i}$ for the regulated packages in the actual allocation and in the allocation without each buyer. Exposure sums the additional regulated orders selected in these exclusion comparisons:

$$\chi_n(q,\omega,\tau^{\text{pol}}) = \sum_i m^{\text{sold}}{}_{\mathcal{R}}^{-i} - (N-1)m^{\text{sold}}{}_{\mathcal{R}} = \sum_i \sum_{j\in\mathcal{R}\setminus\{i\}} (k_j^{-i} - k_j^*) \tag{30}$$

An order can enter several buyers' transfers and counts in each comparison where it does, so its weight in payments differs from the physical quantity that regulated buyers acquire.

Theorem 2.10 (The Exact Exposure Identity)**.** *For fixed q, n, ω, the seller's gross receipts are continuous and piecewise affine in τ^{pol}. Away from their finitely many breakpoints,*

$$\partial_{\tau^{\text{pol}}} R_n = -\bar{q}\chi_n, \qquad 0 \le \chi_n \le \sum_i k_i^* \le n \tag{31}$$

Let $\bar{\chi}_n(q,\tau^{\text{pol}})$ denote expected exposure of payments to regulation under the fixed information law ν_ω. Then

$$W_n(q,\tau^{\text{pol}}) - W_n(q,0) = -\bar{q}\int_0^{\tau^{\text{pol}}} \bar{\chi}_n(q,t)\,dt, \qquad \bar{\chi}_n = \mathbb{E}_{\nu_\omega}\chi_n \tag{32}$$

(see Appendix B.2.3)

Observed purchases therefore do not fully measure how the policy reaches payments. A losing regulated order can price an unaffected winner, so receipts fall with no change in allocation, while a regulated winner can face a payment supported by unaffected demand or retention.

In this fixed-stock, fixed-capacity comparison, expected seller value falls by at most the compliance cost per unit times the physical quantity offered:

$$0 \le W_n(q,0) - W_n(q,\tau^{\text{pol}}) \le n\bar{q}\tau^{\text{pol}} \tag{33}$$

For $n \ge 1$ and $\tau^{\text{pol}} > 0$, define cumulative transmission relative to offered capacity by $\alpha_R = (n\tau^{\text{pol}})^{-1}\int_0^{\tau^{\text{pol}}} \bar{\chi}_n(q,t)dt \in [0,1]$ (see Online Appendix B.1). It is below one if and only if $\int_0^{\tau^{\text{pol}}}[n - \bar{\chi}_n(q,t)]dt > 0$. Relative to actual sales, strict attenuation requires $\int_0^{\tau^{\text{pol}}} \mathbb{E}[m^{\text{sold}} - \chi_n]dt > 0$. In each case unregulated demand or retention must support a positive measure of payment slots in the corresponding comparison.

2.6 Reallocation with Unchanged Extraction

Let $\Xi = (z, n)$ range over the feasible implementation set $\mathcal{X}$, where z is a regional implementation with every complete package offered, so $n = \lfloor q(z)/\bar{q} \rfloor$, with regional cost $C^{\text{impl}}(z)$ and output $q(z)$. Expected profit is

$$\Pi(\Xi, \tau^{\text{pol}}) = \underbrace{W_n(q(z), \tau^{\text{pol}})}_{\text{expected stock value}} - \underbrace{C^{\text{impl}}(z)}_{\text{regional implementation cost}} \tag{34}$$

The selective downstream cost leaves the regional primitives and the chosen equilibrium graph fixed, and the armed group still chooses village coverage, prices, input margins, coercion and residual stock. For each attainable output, the attained cost envelope $\mathcal{C}(q)$ gives the least cost among these regional choices. Procurement expenditure, net import trading terms, transport and coercion enter once, through regional cost (see Online Appendix G.9).

Let Ξ_0 be an attained baseline maximizer and $\Delta_0(\Xi) = \Pi(\Xi_0, 0) - \Pi(\Xi, 0)$ its profit advantage over a feasible competitor before regulation. After the policy, that advantage equals the initial gap less the original choice's additional policy loss relative to the competitor, with exposure evaluated at each choice's own stock and offered capacity.

Theorem 2.11 (Global Output Resilience)**.** *For every feasible competitor Ξ and policy τ^{pol},*

$$\Pi(\Xi_0, \tau^{\text{pol}}) - \Pi(\Xi, \tau^{\text{pol}}) = \Delta_0(\Xi) - \bar{q} \int_0^{\tau^{\text{pol}}} [\bar{\chi}(\Xi_0, t) - \bar{\chi}(\Xi, t)]\, dt \tag{35}$$

The baseline remains optimal exactly when the right-hand side is nonnegative for every feasible competitor. If it is strictly positive for every competitor with a different output, baseline physical output remains uniquely optimal. This conclusion permits multiple cost-minimizing nominal implementations of the same output (see Appendix B.3.1).

On the selling side, the gate reallocates sales when affected buyers bid less, and the value of its original stock is protected if unaffected competing bids or retention orders set every critical payment throughout the policy change.

Even when affected bids set payments and receipts fall, the original choice can remain optimal. Net of Selva's implementation costs, each alternative (other quantities, other offered capacities and exit) must either lose at least as much or start with a profit disadvantage that covers the difference in losses. A positive acquisition margin is not enough, because every package can sell above acquisition cost while another feasible choice remains more profitable.

Corollary 2.12 (Protection by Unaffected Payment-Setting Orders)**.** *If $\bar{\chi}(\Xi_0, t) = 0$ for almost every $t \in [0, T^{\text{pol}}]$, the baseline remains globally optimal throughout that interval. A unique baseline remains unique. If all baseline maximizers have the same output and at least one is protected, that output remains uniquely optimal (see Appendix B.3.1).*

Complete protection requires unaffected demand or retention to set every critical payment almost surely over the policy interval; a high unaffected floor alone still lets regulated

orders set higher payments and pass the shock on. A loss common to nearby choices leaves their ranking unchanged (see Online Appendix G.10).

Corollary 2.13 (Falling Rents with Unchanged Extraction)**.** *Suppose Ξ_0 uniquely maximizes baseline profit, a set $\mathcal{X}^{\text{eq}}$ contains Ξ_0, and $\bar{\chi}(\Xi, t) = \kappa(t)$ for all $\Xi \in \mathcal{X}^{\text{eq}}$ and almost every $t \in [0, T^{\text{pol}}]$. If (35) is strictly positive for all $\Xi \notin \mathcal{X}^{\text{eq}}$ throughout the interval, the baseline choice is unchanged and its profit falls by $\bar{q} \int_0^{\tau^{\text{pol}}} \kappa(t)dt$. If there is a uniform outside gap $\delta^{\text{gap}} = \inf_{\Xi \notin \mathcal{X}^{\text{eq}}} \Delta_0(\Xi) > 0$ and a bounded package count $\bar{Q}$, it is sufficient to restrict policy to*

$$0 \leq \tau^{\text{pol}} < \min\{T^{\text{pol}}, \delta^{\text{gap}}/(\bar{q}\bar{Q})\} \tag{36}$$

(see Appendix B.3.1)

With continuous output, nearby quantities can have arbitrarily close profits, and equal exposure preserves their ranking by imposing the same receipt loss on each. The global comparison handles more distant quantities, exit and alternative village networks. Global regulation can then lower the intermediary's rent while its preferred production choice stays the same (see Online Appendix B.2 and G.11).

2.7 When Regulation Changes Production

When payment exposure varies with inventory, the illegal group can adjust output in response to policy. I consider a fixed smooth feasible regime with a regular interior optimum $z^*(\tau^{\text{pol}})$ and negative definite Hessian $\mathcal{H} = \Pi_{zz}$ in independent feasible coordinates, and let $\boldsymbol{g}_q = \nabla_z q \neq 0$. On this branch, policy enters profit only through $W_n(q, \tau^{\text{pol}})$.

The local response combines how regulation changes the value of an additional unit of physical stock with the regional response. Against a levy, the positive factor Λ measures the regional adjustment and γ the additional payment exposure that comes with additional stock, both at the same zero-policy optimum with capacity and the smooth regime held fixed.

Theorem 2.14 (Local Production Transmission)**.** *At such an optimum,*

$$\frac{dq^*}{d\tau^{\text{pol}}} = -W_{q\tau^{\text{pol}}} \boldsymbol{g}_q{}^{\mathsf{T}} \mathcal{H}^{-1} \boldsymbol{g}_q \tag{37}$$

Thus $W_{q\tau^{\text{pol}}} < 0$ implies falling extraction. Where expected exposure is differentiable, define $\Lambda = -\boldsymbol{g}_q{}^{\mathsf{T}} \mathcal{H}^{-1} \boldsymbol{g}_q > 0$ and $\gamma = \bar{q}\partial_q \bar{\chi}_n$. Against a direct per-unit levy evaluated at the same zero-policy baseline,

$$(dq^*/d\tau^{\text{pol}})_0 = -\Lambda\gamma, \qquad (dq^{\text{levy}}/dt)_0 = -\Lambda \tag{38}$$

Strictly partial local contraction follows when $0 < \gamma < 1$; zero local transmission follows when $\gamma = 0$ (see Appendix B.3.2).

The level bound $0 \leq \bar{\chi}_n \leq n$ places no restriction on $\partial_q \bar{\chi}_n$, the derivative that governs the response. Buyer distributions and retention technology determine its sign and size.

With one offered package, γ is the product of three factors, scaled by package size $\bar{q}$. The first is the fall in the retention reserve with stock, $-\rho'(q)$. The second is the probability that unaffected demand exceeds that reserve, $1 - F_U(\rho(q))$. The third is the regulated-demand density at the threshold, $f_R(\rho(q) + \bar{q}\tau^{\text{pol}})$. Values are measured per package.

Corollary 2.15 (An Analytical Partial-Transmission Condition)**.** *Consider one offered package and independent unaffected and regulated unit-demand values V_U, V_R. Assume F_U is continuous at the relevant reserve values. Let F_U be the unaffected distribution and f_R a continuous regulated density. On an inventory interval with differentiable reserve $\rho(q)$,*

$$\gamma = -\bar{q}\rho'(q)[1 - F_U(\rho(q))]f_R(\rho(q) + \bar{q}\tau^{\text{pol}}) \tag{39}$$

Concave retention makes this weakly nonnegative. A strictly decreasing reserve, positive rival probability and density, and a product in (39) smaller than one give strictly partial local production transmission under the regular regional optimum conditions (see Appendix B.3.2).

Different adjustments can therefore occur at different policy levels. Actual allocation determines who receives mineral, counterfactual allocation determines who sets its price, and a regional switch changes production, coverage or local implementation. Residual stock can also vary continuously while Q stays fixed (see Online Appendix B.6).

2.8 Maximal Reach of Global Policies

I consider a global demand-side policy that excludes all regulated buyers while leaving unaffected buyers and retention outlets available. This takes the compliance tariff to infinity, $\tau^{\text{pol}} \to \infty$. Unaffected buyers can still buy gold, and the illegal group can still launder or store its stock. The buyer population, information law, retention technology and regional implementation opportunities stay fixed, while production and offered capacity may adjust.

At fixed stock q and offered capacity n, let $p_i^{\mathcal{U}}$ be a remaining buyer's payment in the auction without regulated buyers, $y_n^{\mathcal{U}}$ the stock the gate retains and $R_n^{\mathcal{U}}$ seller value. The receipt floor is

$$\underline{W}_n(q) = \mathbb{E}_{\nu_\omega} R_n^{\mathcal{U}} = \underbrace{\mathbb{E}_{\nu_\omega} \sum_{i \in \mathcal{U}} p_i^{\mathcal{U}}}_{\substack{\text{payments from} \\ \text{remaining buyers}}} + \underbrace{\mathbb{E}_{\nu_\omega} H(y_n^{\mathcal{U}})}_{\substack{\text{net value of} \\ \text{retained stock}}} \geq H(q) \tag{40}$$

Laundering and storage also support auction payments through reservation orders, an effect already included in buyer transfers. At fixed stock and offered capacity, falling regulated bids weakly raise retained stock and, when retention value is nonnegative and receipts are positive, the share of receipts that retained uses supply (Corollary A.19); the increase can disappear once stock or capacity changes (see Online Appendix G.12).

With all complete packages offered, $\underline{W}(q) = \underline{W}_Q(q)$ with $Q = \lfloor q/\bar{q} \rfloor$. I normalize exit profit to zero and assume that all profit maxima are attained. The production floor requires the remaining outlets to cover Selva's full implementation cost and leave a strictly positive surplus:

$$\underline{\Pi} = \max \left\{ 0, \max_{q>0 \text{ feasible}} \left[\underbrace{\underline{W}(q)}_{\substack{\text{remaining buyers} \\ \text{and retained uses}}} - \underbrace{\mathcal{C}(q)}_{\text{implementation cost}} \right] \right\} > 0 \tag{41}$$

Suppose also that every active implementation costs at least $c^{\text{act}}_{\min} > 0$ and total stock value is at most $\bar{v}q$, with the same $c^{\text{act}}_{\min}$ and $\bar{v} > 0$ across all feasible outputs, coverage choices, offered capacities and tariffs. Let $q_0 > 0$ denote baseline output. These conditions apply to the following result.

Theorem 2.16 (Global Policy Ceiling)**.** *Under fixed buyer assumptions, the global policy cannot reduce receipts and quantity below the following floors:*

$$\textit{Receipt floor:} \qquad W(q, \tau^{\text{pol}}) \geq \underline{W}(q), \tag{42}$$

$$\textit{Production floor:} \qquad q_{\tau^{\text{pol}}} \geq \frac{c^{\text{act}}_{\min} + \underline{\Pi}}{\bar{v}} > 0, \qquad \frac{q_{\tau^{\text{pol}}}}{q_0} \geq \frac{c^{\text{act}}_{\min} + \underline{\Pi}}{\bar{v} q_0} > 0 \tag{43}$$

The receipt floor reflects the value of unaffected demand and the option to launder or store gold. The production floor follows because the gate must produce enough to cover its costs and earn the profit these surviving outlets still offer (see Appendix B.4).

At fixed stock and capacity, the largest receipt loss is $W_n(q, 0) - \underline{W}_n(q)$, a supremum when the floor is approached only as $\tau^{\text{pol}} \to \infty$ (Proposition Q.2). Let $\Pi^*(\tau^{\text{pol}})$ denote optimized profit. Its loss is at most $\Pi^*(0) - \underline{\Pi}$, a bound attained when regulated marginal values share a finite common upper bound (Proposition Q.3).

Relative to baseline output, proportional contraction cannot exceed $1 - (c^{\text{act}}_{\min} + \underline{\Pi})/(\bar{v} q_0) < 1$ (Proposition Q.4), and the bound holds at every policy optimum (see Online Appendix B.5.1).

Under the conditions of Theorem 2.16, output remains positive even when a global policy excludes every regulated buyer but leaves unaffected buyers and retention outlets available. Unaffected demand and retention leave a profitable outlet. The next section asks when recorded origin becomes a credible condition of access to certified buyers.

3 Blockchain as a Certification Device

Theorem 2.16 sets a ceiling on a policy that excludes regulated buyers while leaving other outlets available; the metal does not reveal its origin. Certification conditions access to certified buyers on an origin claim. An organizer can profit by obtaining approval for illegal material, but needs the cooperation of those who control admission. Later discovery can exclude the material from certified trade and trigger losses that the institution can collect.

3.1 Physical Backing, Ancestry and Exclusion

A claim records a physically identified quantity of gold. A live claim has not been consumed or retired; it may nevertheless be ineligible for certified settlement. An atomic transition consumes input claims and creates output claims in the same recorded step.

To turn false admission and discovery into physical outcomes, the record must preserve backing and ancestry as owners change. Let μ^{mass} measure recoverable mineral content. A live claim c^{cl} records its owner, custody evidence, positive mass $v(c^{\text{cl}})$ and physically identified backing $\psi(c^{\text{cl}})$ with $\mu^{\text{mass}}(\psi(c^{\text{cl}})) = v(c^{\text{cl}})$. Live claims have disjoint backing. Consider a processor that turns live inputs I^{cl} into fresh disjoint outputs O^{cl}, with recorded loss $m_{\text{loss}} \geq 0$, input mass $v(I^{\text{cl}})$ and combined backing $\psi(I^{\text{cl}})$. It must satisfy

$$v(O^{\text{cl}}) + m_{\text{loss}} = v(I^{\text{cl}}), \qquad \psi(O^{\text{cl}}) \subseteq \psi(I^{\text{cl}}), \qquad m_{\text{loss}} \geq 0 \tag{44}$$

Recorded loss cannot back another claim, and redemption retires a claim. Inspection and custody maintain the physical facts. Cryptography authenticates the records of those facts. Event and derivation standards can represent the links [GS1, 2023, Moreau and Missier, 2013].

Let S_t be live claims after settlement event t and $\text{Anc}(c^{\text{cl}})$ a claim's recorded origin admissions, which outputs inherit from inputs (see Online Appendix H.16). Let $\mathcal{O}_t^{\text{rev}}$ and $\mathcal{O}_t^{\text{susp}}$ be the origin admissions under effective revocation and suspension. Their recorded descendants lose eligibility. A claim is eligible only if none of its recorded origins is excluded:

$$C_t = \{c^{\text{cl}} \in S_t : \text{Anc}(c^{\text{cl}}) \cap (\mathcal{O}_t^{\text{rev}} \cup \mathcal{O}_t^{\text{susp}}) = \varnothing\} \tag{45}$$

Exclusion changes eligibility and preserves history. Intake checks keep excluded backing from returning under a new identifier until authorized reinstatement. A lot whose affected portions cannot be separated is excluded whole (see Online Appendix H.15).

Proposition 3.1 (Conservation and Retrospective Exclusion)**.** *Suppose initial claims have disjoint, correctly measured backing and finite genesis histories. Maintain physical identification, complete ancestry, atomic transitions, authenticated status decisions and intake checks against excluded origins. Let A_t be cumulative admitted mass and X_t*

cumulative processing loss and redemption mass. Every live claim then has finite provenance, no positive mass backs two live claims, and

$$v(S_t) = \mu^{\text{mass}}\left(\bigcup_{c^{\text{cl}} \in S_t} \psi(c^{\text{cl}})\right) = v(S_0) + A_t - X_t \tag{46}$$

A status decision alone changes neither side. After effective revocation, the origin's live recorded descendants cannot settle as certified or regain eligibility through a fresh identifier until authorized reinstatement (see Appendix C.1; Online Appendix C.1.1 and C.1.2).

3.2 The Transaction, Participants and Timing

An admission organizer procures a unit of newly mined gold and presents it at an approved mine gateway, where the licensed producer or custodian and assigned inspectors control approvals. In the solved example, inspectors perform the required inspection before reporting and reject illegal backing unless bribed; collusion changes the report without saving the inspection cost. The organizer chooses the backing and knows its forgery cost. In the solved market, a downstream buyer sees the contract and recorded origin but observes neither the backing nor that cost. The organizer and the gate of Sections 1–2 can be different owners. The example holds extraction fixed and starts from acquired illegal stock, so its gain is an admission return; Section 4 considers the gate's receipts.

The buyer pays an invoice with a commodity component for procurement and a prepaid fee for inspection, records and enforcement capacity. Liable participants post bonds that an adverse finding can forfeit along with certification. The organizer needs cash or pledgeable credit to pay bribes before fraudulent receipts arrive. Buyers pay for their own investigation and bear the losses the contract assigns them (see Online Appendix H.2). The sequence is:

1. The contract specifies fees, bonds, participation terms and the claims horizon. Buyers accept and escrow their invoices; service providers accept their contracts and liable agents post funded security.

2. The organizer chooses backing and offers any admission bribes. Required inspectors perform the inspection and submit their reports.

3. Admitted units are allocated and commodity payments released. Buyers can investigate or hire an inspector, insurer or specialist.

4. A holder of actionable evidence decides whether to disclose it. Opportunities to contract over concealment differ between the general capture analysis and the solved market, as specified below.

5. The authority adjudicates, changes eligibility and collects sanctions. Security is released or reused only after the associated exposure closes or replacement finance is secured.

New extraction enters certified trade only at approved mines from launch. Later custody, processing and recycling inherit their predecessors' records and status, so none supplies a fresh clean origin (see Online Appendix H.1).

3.3 Why a Purchaser Investigates and Reports

Verification uses resources [Townsend, 1979], and who pays affects what is reported, as in the client-paid audits of Duflo et al. [2013]. With beliefs fixed and no side payment, consider purchaser j. Relative to an otherwise identical eligible purchase, let $L_{E,j}$ be its currency loss after its own early finding and report, $L_{L,j}$ its loss after later discovery elsewhere and $L_{U,j}$ its loss if the defect stays undiscovered during the horizon. Let ${P^{\text{disc}}}_{-j}$ be the probability of discovery through other channels over the remaining horizon (see Online Appendix H.4). At fixed quantity and price, the expected loss avoided by reporting early is

$$ {A^{\text{disc}}}_j = {P^{\text{disc}}}_{-j} L_{L,j} + (1 - {P^{\text{disc}}}_{-j}) L_{U,j} - L_{E,j} \tag{47} $$

The outcome under silence includes feasible mitigation and any effect of concealment on liability, resale or access to evidence. Before learning the truth, the purchaser chooses effort $e_j \geq 0$. Let π_j be its posterior probability of illegal origin, $s_j(e_j)$ the probability of conclusive early evidence given fraud, and $c_j(e_j)$ its currency investigation cost. I assume $s_j(0) = c_j(0) = 0$, no false positives, and early evidence independent of other channels given fraud and the public state. Effort pays only when fraud is present, evidence is obtained and reporting avoids ${A^{\text{disc}}}_j$, so its gain over zero effort is

$$ \pi_j s_j(e_j) {A^{\text{disc}}}_j - c_j(e_j) \tag{48} $$

Proposition 3.2 (Investigation and Voluntary Disclosure)**.** *After conclusive evidence, reporting is optimal exactly when* ${A^{\text{disc}}}_j \geq 0$*, with indifference at equality. When reporting is chosen, the investigation gain is* (48)*. Suppose* ${A^{\text{disc}}}_j > 0$*, the feasible effort interval is compact, both primitives are continuous,* s_j *is increasing and concave, and* c_j *is increasing and strictly convex. Optimal effort is unique. With differentiable primitives, an interior choice equates the marginal investigation cost to its expected loss reduction:*

$$ c_j'(e_j) = \pi_j {A^{\text{disc}}}_j s_j'(e_j) \tag{49} $$

(see Appendix C.2; full derivation in Online Appendix C.1.4)

If early and late discovery impose the same loss $L > 0$ and undiscovered fraud imposes none, then ${A^{\text{disc}}}_j = -(1 - {P^{\text{disc}}}_{-j})L$. With uncertain later discovery, investigation then accelerates a loss the buyer might avoid, and liability alone does not induce diligence (see Online Appendix H.5). At zero fraud risk, investigation with no separate benefit has no private return. Scrutiny under complete deterrence therefore requires paid inspection, another valuable service or the separately specified off-path environment.

3.4 Finite Diversion and the Funded Cost of Capture

For one unit, let h be the currency cost of clean procurement, p_D the committed outside receipt for an already acquired illegal unit and c^{forg} the extra forgery cost. Substitution saves h, forgoes p_D and incurs c^{forg}, so before bribes and sanctions its gain is

$$G(c^{\text{forg}}) = \Gamma^{\text{adm}} - c^{\text{forg}}, \qquad \Gamma^{\text{adm}} = h - p_D \tag{50}$$

The sunk acquisition payment cancels, and the solved market uses this linear case. With several batches or a nonlinear selling continuation, the organizer values its entire remaining stock in a fixed market state Ω. That state includes the original stock, history and committed selling mechanism. Let q be its illegal stock and a^{div} a feasible joint action of fraudulent batches, custody paths and false approvals that diverts mass $d(a^{\text{div}})$ with $0 < d(a^{\text{div}}) \leq q$. Let $F_C(a^{\text{div}}; \Omega)$ be certified receipts net of maintained ordinary payments. Let $k_f(a^{\text{div}}; \Omega)$ be additional physical and concealment costs, excluding side payments and sanctions. Let $W_D(y; \Omega)$ be the committed continuation value of y illegal units. The gain is

$$G_\Omega(a^{\text{div}}) = F_C(a^{\text{div}}; \Omega) - k_f(a^{\text{div}}; \Omega) - [W_D(q; \Omega) - W_D(q - d(a^{\text{div}}); \Omega)] \tag{51}$$

Both continuation terms hold the mechanism and information fixed, preserving Section 2's distinction between a committed rule and a new rule chosen after diversion (see Online Appendix H.6). Side agreements change the reports on which admission and enforcement depend [Tirole, 1986]. Let $\mathcal{S}^{\text{app}}$ contain the organizer, indexed by 0, and every original participant whose approval a^{div} requires. Let $\mathcal{J}^{\text{rep}}$ be the finite, disjoint set of buyers, insurers or investigators that can obtain and report actionable evidence and whose silence can be bought. Holders outside $\mathcal{J}^{\text{rep}}$ enter residual discovery and the losses $a_i(\mathcal{T}^{\text{sil}})$. I assume that the organizer knows these channels and their reservation payoffs. Approval and silence agreements are jointly enforceable over the horizon. Parties are risk neutral, retain ordinary earnings, have no omitted fraud benefit and accept at equality. Payments compensate actual collectible losses or disclosure alternatives, and a paid holder cannot reopen the bargain or report anyway (see Online Appendix H.7).

For a silence set $\mathcal{T}^{\text{sil}} \subseteq \mathcal{J}^{\text{rep}}$, original participant i faces expected incremental collectible loss $a_i(\mathcal{T}^{\text{sil}}) \geq 0$ relative to honesty. A silenced reporter j requires at least $\sigma_j(\mathcal{T}^{\text{sil}}) \geq 0$ delivered currency units relative to disclosure. A gross payment delivers the fraction $\lambda_i \in (0, 1]$ to its recipient, and losses and reservation payoffs remain unchanged by transfers above the minimum. The advance bill covers approvers other than the organizer and every purchased silence agreement:

$$\mathcal{B}^{\text{adv}}(\mathcal{T}^{\text{sil}}) = \sum_{i \in \mathcal{S}^{\text{app}} \setminus \{0\}} \frac{a_i(\mathcal{T}^{\text{sil}})}{\lambda_i} + \sum_{j \in \mathcal{T}^{\text{sil}}} \frac{\sigma_j(\mathcal{T}^{\text{sil}})}{\lambda_j} \tag{52}$$

Let $F_0^{\text{adv}}(a^{\text{div}}, \Omega)$ be the cash or enforceable credit available before settlement after other committed advances. Future fraudulent receipts count only when pledgeable. The organizer

chooses the cheapest affordable silence plan and bears its own expected loss, with value $+\infty$ when no plan is affordable:

$$K_J^F(a^{\text{div}}) = \min_{\substack{\mathcal{T}^{\text{sil}} \subseteq \mathcal{J}^{\text{rep}} \\ \mathcal{B}^{\text{adv}}(\mathcal{T}^{\text{sil}}) \leq F_0^{\text{adv}}(a^{\text{div}}, \Omega)}} \{a_0(\mathcal{T}^{\text{sil}}) + \mathcal{B}^{\text{adv}}(\mathcal{T}^{\text{sil}})\} \tag{53}$$

Theorem 3.3 (Funded Capture with Optional Suppression)**.** *Under this protocol, action a^{div} has a strictly profitable funded implementation if and only if $G_\Omega(a^{\text{div}}) > K_J^F(a^{\text{div}})$. Thus no profitable funded diversion exists if $G_\Omega(a^{\text{div}}) \leq K_J^F(a^{\text{div}})$ for every feasible nonzero joint action. If $G_\Omega(a^{\text{div}}) < K_J^F(a^{\text{div}})$ for every feasible nonzero joint action, indifferent implementations are excluded as well.*

Cooperation must leave a surplus and its advances must be affordable (see Appendix C.3; Online Appendix H.8). Future liability reduces profit, but cannot finance a bribe (see Online Appendix C.1.7).

3.5 Independent Evidence and the Limits of Suppression

An additional investigator matters when it adds a complete sequence of usable evidence, disclosure and effective adverse decision. For a fixed admitted fraud, let r_0 be baseline effective discovery by the end of the claims horizon. Let r_j be the conditional probability of that sequence through channel j. Under conditional independence, survival requires failure in every channel:

$$P^{\text{disc}}{}_n = 1 - (1 - r_0) \prod_{j=1}^{n} (1 - r_j) \tag{54}$$

An additional channel contributes $(1 - P^{\text{disc}}{}_n) r_{n+1}$ to discovery. Adding ledger participants does not itself create new evidence. Splitting fixed effort among more reviewers need not improve discovery, and independent efforts cannot remove a common source or adjudication failure (Online Appendix C.1.5, H.9 and H.10). Holding action, gain and advance cap fixed, adding a channel weakly raises the funded capture cost when each expanded funded plan implies an affordable old plan at weakly lower total cost. Suppression costs stay bounded in the homogeneous-channel calculation. There, advance finance does not bind, and the organizer can stop buying silence and bear the bounded collectible loss. With a binding advance cap that plan may be unavailable, and (53) governs the choice (Online Appendix, Propositions C.5 and C.6; see also H.11 and H.12).

3.6 A Posted-Contract Equilibrium of Fraud and Scrutiny

The solved market uses (50) to link a buyer's beliefs to investigation and back to fraudulent admission. There is a unit mass of equal-sized certification slots for an origin exposed to fraudulent substitution. Clean suppliers serve these slots at quote h, which covers their costs and reservation returns. Each organizer holds one acquired illegal unit with committed outside receipt p_D. Its privately observed forgery cost c^{forg} is uniform on

$[0, \bar{c^{\text{forg}}}]$, with currency cost bound $\bar{c^{\text{forg}}} > 0$. Buyers are randomly assigned, independently of c^{forg}, and admitted labels pool.

Amounts are per unit of slot mass. The organizer posts deposit B_0 and required participant i posts b_i, giving collectible security $D = B_0 + \sum_i b_i > 0$. The security is reserved for this slot until its claims horizon closes; no other slot or unresolved vintage can draw on it. Side transfers deliver face value, so $\lambda_i = 1$. Separate advance funds $F_{\text{adv}} \geq \sum_i b_i$ make every minimum bribe schedule affordable. The buyer escrows $h + \phi$, where ϕ prepays ordinary verification and continuing services. It has liquidity for the invoice and worst contractual loss and finances its own investigation. Honest wages cover effort, bond funding and reservation returns. Ordinary inspection is paid under either backing, and displaced clean procurement has no unpaid claim (see Online Appendix H.13).

Before evidence, buyers cannot agree to suppress investigation or make other side contracts. After verifiable evidence, a buyer can bargain over silence with the whole original coalition, which can cofinance an enforceable agreement. The finding does not notify the authority, and an independent late-enforcement channel stays beyond suppression. The stated bonds and buyer losses exhaust the modeled appropriable rewards and liabilities. A buyer with fraud belief π chooses effort $e \in [0, 1]$ at currency cost $\kappa e^2/2$, with $\kappa > 0$. Given fraud, it obtains conclusive early evidence with probability e, without false positives. Reporting causes incremental loss $L_E \geq 0$, which requires documented disclosure. Silence leaves late discovery with probability $r^{\text{late}} \in (0, 1)$ and loss $L_L > 0$, net of feasible silent mitigation. Undiscovered fraud costs nothing further within the horizon (see Online Appendix H.14).

After a finding, reporting avoids expected losses of $A^{\text{disc}} = r^{\text{late}} L_L - L_E$, so the buyer requires at least A^{disc} to give up reporting. The coalition saves at most $(1 - r^{\text{late}})D$ in expected bond forfeiture, since disclosure forfeits D and silence leaves expected forfeiture $r^{\text{late}} D$. I impose

$$A^{\text{disc}} > (1 - r^{\text{late}})D \tag{55}$$

Every genuine finding, on or off path, is then reported, even though larger bonds that deter fraud also raise what the coalition would pay for silence. Before learning, the buyer pays for effort and bears the early reporting loss or the expected late loss on undetected fraud. Its additional expected cost is

$$\mathcal{L}^{\text{buy}}(\pi, e) = \pi r^{\text{late}} L_L - \pi A^{\text{disc}} e + \frac{\kappa e^2}{2} \tag{56}$$

Minimization gives $e = \min\{A^{\text{disc}} \pi/\kappa, 1\}$. Discovery happens early or through late enforcement, so $P^{\text{disc}}(e) = r^{\text{late}} + (1 - r^{\text{late}})e$. Participant i needs advance compensation $P^{\text{disc}}(e) b_i$. With the organizer's own expected bond loss, the capture cost is $P^{\text{disc}}(e)D$. Fraud is then profitable for $c^{\text{forg}} < \Gamma^{\text{adm}} - P^{\text{disc}}(e)D$, with ties choosing honesty. Buyers observe neither c^{forg} nor backing, so by Bayes' rule their pooled belief equals the fraud fraction, which negligible agents take as given. Let $\alpha^{\text{scr}} = A^{\text{disc}}/\kappa$ be the buyer's effort response coefficient and $\widetilde{\Gamma^{\text{adm}}} = \Gamma^{\text{adm}} - r^{\text{late}} D$ the substitution margin after baseline expected sanctions. The

restrictions below keep that margin between zero and the forgery-cost range and keep effort interior.

Theorem 3.4 (Positive-Trade Interior Equilibrium)**.** *Maintain* (55) *and suppose*

$$0 < \widetilde{\Gamma^{\text{adm}}} < \bar{c^{\text{forg}}}, \qquad \alpha^{\text{scr}} > 0, \qquad \alpha^{\text{scr}}\widetilde{\Gamma^{\text{adm}}} < \bar{c^{\text{forg}}} + D(1 - r^{\text{late}})\alpha^{\text{scr}} \tag{57}$$

Subject to buyer participation, the market has a unique fraud fraction and effort choice:

$$\pi^* = \frac{\widetilde{\Gamma^{\text{adm}}}}{\bar{c^{\text{forg}}} + D(1 - r^{\text{late}})\alpha^{\text{scr}}}, \qquad e^* = \alpha^{\text{scr}}\pi^*, \qquad P^{\text{disc}*} = r^{\text{late}} + (1 - r^{\text{late}})e^* \tag{58}$$

All slots trade, $0 < \pi^* < 1$ *and* $0 < e^* < 1$*. Required participants accept their minimum compensating bribes when offered; buyers disclose actionable evidence. Honest services, advance payments and continuing obligations are funded.*

A higher expected fraud rate induces investigation. Investigation in turn makes fraud profitable at fewer forgery costs. With buyer service value v, $v > h + \phi + \mathcal{L}^{\text{buy}}(\pi^*, e^*)$ suffices for unit demand at the posted invoice (see Appendix C.4; full derivation in Online Appendix C.1.11).

3.7 Surviving Certified Gold

Removing only the early evidence opportunity from the same posted contracts, with all other primitives fixed, gives $e_0 = 0$ and $\pi_0 = \widetilde{\Gamma^{\text{adm}}}/\bar{c^{\text{forg}}}$. Continuing scrutiny gives $\pi^* < \pi_0$ and $P^{\text{disc}*} > r^{\text{late}}$. Let Q^{adm} be admitted cohort mass and U its illegal component. Let R_U and R_A be the illegal and admissible mass later withdrawn. With no other entries or exits, admitted and surviving contamination are

$$\iota^{\text{in}} = \frac{\mathbb{E}U}{\mathbb{E}Q^{\text{adm}}}, \qquad \iota^{\text{live}} = \frac{\mathbb{E}(U - R_U)}{\mathbb{E}(Q^{\text{adm}} - R_U - R_A)} \tag{59}$$

For equal-unit admissions with fraud fraction π, illegal-only withdrawal probability P^{disc} and no other exits, surviving illegal mass is $\pi(1 - P^{\text{disc}})$ and live certified mass is $1 - \pi P^{\text{disc}}$, so

$$\iota(\pi, P^{\text{disc}}) = \frac{\pi(1 - P^{\text{disc}})}{1 - \pi P^{\text{disc}}} \tag{60}$$

In the matched comparison, continuing scrutiny lowers the ratio (see Online Appendix H.17). Selective removal weakly lowers contamination under the stated conditions (see Online Appendix D.1). Aggregation across countries must use true extraction origin. For origin k, let V_k be surviving certified mass and U_k its illegal component. Then $U_k = 0$ outside

the set $\mathcal{O}^{\text{ill}}$ of origins with illegal extraction in the cohort. World contamination weights each origin by its share of expected live certified mass:

$$\begin{aligned} \iota_{\text{world}}^{\text{live}} &= \frac{\sum_k \mathbb{E}U_k}{\sum_k \mathbb{E}V_k} = \sum_{k\in\mathcal{O}^{\text{ill}}} w_k^{\text{live}} \iota_k^{\text{live}}, \quad w_k^{\text{live}} = \frac{\mathbb{E}V_k}{\sum_h \mathbb{E}V_h} \\ \iota_k^{\text{live}} &= \frac{\mathbb{E}U_k}{\mathbb{E}V_k}, \qquad\qquad\qquad\qquad w_k^{\text{in}} = \frac{\mathbb{E}Q_k^{\text{adm}}}{\sum_h \mathbb{E}Q_h^{\text{adm}}} \end{aligned} \tag{61}$$

Here Q_k^{adm} is admitted mass from true origin k (see Online Appendix H.18). Admission weights govern expected purchase costs; live weights reflect the composition after withdrawal and can differ. A purchaser in a country with no illegal extraction can still buy material whose true origin lies in $\mathcal{O}^{\text{ill}}$. In the posted-contract example, each organizer either admits its acquired illegal unit fraudulently or sells it outside, so outside deliveries of illegal stock are $1-\pi$. Better certification can therefore increase them at unchanged illegal extraction by reallocating acquired stock (see Online Appendix D.2).

3.8 Financing Credibility and Measuring Its Cost

Honest providers must cover costs and reservation returns even under complete deterrence. Due diligence requires records, monitoring, supplier adjustments and audits [OECD, 2021a]. Fees and committed outside funds finance admission, custody, records, forensic work, adjudication, availability, capital costs and promised obligations. Finance must last through exposure; a common bond is counted once across unresolved claims and cannot be released early without replacement funds (see Online Appendix H.19).

Prevention costs recur even under complete deterrence. Receipts that disappear when no fraud is discovered cannot finance them alone. With bounded fees, bounded committed outside funding and a positive lower bound on verification cost per unit, funded volume is bounded (see Online Appendix C.1.10). Expanding certified trade therefore requires a larger funding base or more productive verification (see Online Appendix H.20).

Let ϕ_0 be the baseline service fee and $\Delta\phi > 0$ the extra record, capacity and exposure cost of early scrutiny. Fees equal service and capital opportunity costs. For this resource comparison, the private losses L_E and L_L consist of legal, replacement-logistics and lost-service costs; the invoice and damages transfers are not counted again. The baseline has expected incident loss $\pi_0 r^{\text{late}} L_L$, and scrutiny gives optimized buyer cost $\mathcal{L}^{\text{buy}}(\pi^*, e^*)$. Total institutional and downstream resource costs are

$$\mathcal{C}_0 = \phi_0 + \pi_0 r^{\text{late}} L_L, \qquad \mathcal{C}_1 = \phi_0 + \Delta\phi + \mathcal{L}^{\text{buy}}(\pi^*, e^*) \tag{62}$$

Costs rise if and only if $\Delta\phi > \pi_0 r^{\text{late}} L_L - \mathcal{L}^{\text{buy}}(\pi^*, e^*)$, and more prevention alone does not determine that sign (see Online Appendix H.21). An exact illustration in Online Appendix D.4 shows costs rising while live contamination falls.

3.9 From Certification to Production and Government Action

Certification changes trading opportunities according to origin, while fraud, scrutiny and funding remain costly. The admission return on acquired stock is not the illegal group's extraction profit. Section 4 brings these opportunities into world market clearing to determine their effects on prices and research returns; Section 5 examines government intervention.

4 Global Effects of Certification on Prices and Directed Innovation

Directed technical change predicts that research follows the larger and more profitable market [Acemoglu, 2002, Acemoglu et al., 2012]. In Acemoglu et al. [2012] clean and dirty inputs are distinct technologies. Legal and illegal gold are the same metal, so before origin can be verified there is a single market to follow. A credible record of origin creates two, each with its own price. Whether research then moves toward legal mining depends less on the price of illegal gold than on what an inventor keeps. Certified innovation is Schumpeterian: an inventor serving legal mines patents an improved machine design and licenses it until a rival's invention displaces it, as in Aghion and Howitt [1992]. The armed group cannot register or enforce a patent: an improvement at its mines gives its owner one production interval of advantage and then becomes common knowledge. The asymmetry holds under pooling and under certification: certification changes access to output markets, not patent rights.

Relative to directed-technical-change models of the environment, including their North–South versions [Acemoglu et al., 2012, 2014, Hémous, 2016, Witajewski-Baltvilks and Fischer, 2023], two features distinguish this setting. The clean and dirty goods are the same metal, separated only by an institution that records origin. The asymmetry in what inventors keep has an institutional cause: an illegal operator cannot hold a patent. The section then connects directed innovation to the fiscal value of a legal mine, which Section 5 turns into the decision to retake a mining region.

The section answers three questions. Does certification lower the price of uncertified gold? Does a lower price lower the reward to uncertified research? When does accumulated certified technology make certified research privately sustainable? The answers give Section 5 its two inputs: the income of a mine the illegal group still controls and the fiscal value of a legal one. Two links are left open, and I state them once here. The certification-date comparison of Section 4.1 and the growth economy of Section 4.3 share production and ownership but use different demand systems; I do not derive the transition from one to the other, nor do I embed the partial-fraud allocation of Section 3 in the growth price system (see Online Appendix E.1). A benchmark with one-interval rights in both sectors, independent regional supplies and a fixed world budget compares the takeover cutoff under certification and pooling directly and determines prices jointly with fraud (see Online Appendix L). The destination evidence and the full production and ownership specification are in the Online Appendix (see Online Appendix, Table 13 and Proposition I.1).

4.1 Prices under Pooling and Certification

Before origin is verifiable, every buyer purchases from an anonymous pool with a common clean proportion and cannot choose its own source mix. Once origin is verifiable, North,

the regulated buyers of Section 2, buys certified gold only, while South, the unaffected buyers, chooses between both origins. Let $0 \leq \nu_{NS} < 1$ be North's share of mineral expenditure. For this comparison I hold inherited technology and both blocs' mineral budgets fixed at the certification date, before research changes technology; Section 4.3 lets income grow. Let ϕ_{NS} be the uniform certified charge, so the producer keeps the fraction $1 - \phi_{NS}$ of the certified price. South's demand is CES with elasticity $\varepsilon_{NS} > 1$ and relative weight $\omega_{NS} > 0$ on certified gold. Supply comes from legal and illegal sites that hire mobile workers alongside coerced Selva output. In the formal statements, "dirty" denotes uncertified gold.

Theorem 4.1 (Price Effect of Segmentation)**.** *Consider the mixed-site supply of the Online Appendix, with positive mobile capacity in both sectors, fixed positive available ordinary labor, coerced supply $Z^{D,coer} p_D^{m_\beta - 1}$, and the uniform certified charge $\phi_{NS} \in [0, 1)$ as the only changed production receipt wedge. Write $m_\beta = 1/\beta_m > 1$ and $Q_j = p_D^{m_\beta - 1} q_j(\zeta)$ at the price ratio $\zeta = p_C/p_D$. Let $F(\zeta) = \zeta q_C(\zeta) + q_D(\zeta)$ and let F_0 be total pooled supply at unit quote, with the same inherited technology and charges. There is a unique $\bar{\zeta}$ satisfying $F(\bar{\zeta}) = F_0$, the crossing of the expenditure-weighted supply coefficient. It equals one when $\phi_{NS} = 0$, and lies strictly between 1 and $1/(1 - \phi_{NS})$ when $\phi_{NS} > 0$. Define*

$$\bar{\nu_{NS}} = \frac{\bar{\zeta} q_C(\bar{\zeta})/q_D(\bar{\zeta}) - \omega_{NS} \bar{\zeta}^{1-\varepsilon_{NS}}}{1 + \bar{\zeta} q_C(\bar{\zeta})/q_D(\bar{\zeta})} \tag{63}$$

The dirty price falls if and only if $\nu_{NS} > \bar{\nu_{NS}}$; equality gives no dirty-price change and reversal gives a rise. Whenever the dirty price falls, the certified gross price rises (see Online Appendix I.2).

North's exit from the pool removes its spending from uncertified gold. The uncertified price falls when that loss exceeds what South's substitution and the reallocation of mobile workers between sectors restore. With no certification charge, $\bar{\zeta} = 1$ and the threshold has the closed form

$$\bar{\nu_{NS}} = (1 + \omega_{NS}) s_0 - \omega_{NS} \tag{64}$$

where s_0 is the certified share of pooled spending (see Online Appendix K.6). The uncertified price then falls exactly when North wants a larger share of spending than the certified share of supply can absorb, net of South's own taste for certified gold. A positive charge separates the price buyers pay from the fraction $1 - \phi_{NS}$ producers keep; Section 4.2 gives the condition under which certified producers gain.

The price effect reaches the mine. On the installed-equipment branch of Selva, a coerced operation with fixed capacity, an unchanged procurement coefficient and $\alpha_m = 1 - \beta_m$ supplies output proportional to $p_D^{m_\beta - 1}$ and earns profit proportional to $p_D^{m_\beta}$, with $m_\beta = 1/\beta_m$. Its output and profit therefore fall exactly when $\nu_{NS} > \bar{\nu_{NS}}$ (see Online Appendix K.20). This is the contrast with Theorem 2.16. A rule on buyers lowers regulated bids, but unaffected buyers and laundering at the outside price keep a floor under the armed group's receipts. Certification moves the outside price itself, so the floor falls with it. The effect has a sign condition: when $\nu_{NS} < \bar{\nu_{NS}}$ the uncertified price rises and the

coerced mine gains. On this branch households have no variable mining labor, so the subsistence response of Section 1 is absent. In general Selva with a linear receipt, the armed group's optimal output weakly falls when p_D falls, because its implementation cost does not depend on the outside price. Subsistence shapes that cost, and so the size of the response, not its direction (see Online Appendix K.5).

4.2 Price and Prize

A lower uncertified price need not lower the reward to uncertified research. Compare a project undertaken at date t and first producing at $t+1$ under pooling and under certification. Ceteris paribus, hold inherited technology outside the project, available ordinary labor, company networks, line assignment and ownership rights fixed. Two reversals are then possible (see Online Appendix J.3).

First, the ordinary wage can fall together with the uncertified price. A mobile producer that gains from cheaper labor can then earn a larger operating gain from an uncertified prototype despite the lower price. The coerced mine of Section 4.1 does not hire at that wage: with $\varrho^p = p_D/p_0$, its output scales as $\varrho^{pm_\beta - 1}$ and its operating profit and the gain on a proportional prototype claim as ϱ^{pm_β}. This reversal requires a positive certification charge: with no charge, the uncertified price and the mobile owner's gain cross their pooled values at the same threshold (see Online Appendix K.5). Second, the certified inventor's first operating flow changes by the ratio of certified producers' net spending, $(1-\phi_{NS})E_C^C$, to pooled clean spending, E_C^P: $M_C = (1-\phi_{NS})E_C^C/E_C^P$. That ratio exceeds one only if the certification charge is smaller than the uncertified share of pooled spending, $\phi_{NS} < 1 - s_0$ (see Online Appendix K.1). The first flow is not the whole reward: later patent income and deployment costs also enter, and Appendix D.1 gives the exact cost allowance under which a strictly private pooled allocation stays private after certification. Segmentation alone therefore signs neither the change in research nor the support it needs.

4.3 Growth with an Expanding Certified Frontier

The illegal group's return remains the continuation value of its auction, retention and trading opportunities from Section 2, not a world price times output. A change in a quoted price therefore need not pass one-for-one to its marginal return. The growth application uses a common-quote branch, on which the selling contracts pay a linear marginal receipt; linear retention, $H(y) = p_D y$, with no buyer valuing a marginal package above $p_D\bar{q}$, is a sufficient case (see Online Appendix I.2). On the royalty-only certified branch the producer receives $p_{r,t}^C = (1-\tau_r^G)h_t$, where h_t is the gross certified quote and τ_r^G a public royalty, a payment distinct from the certification fee.

Take $0 < \beta_m < 1$, $\alpha_H, \alpha_L > 0$ and $\alpha_H + \alpha_L < 1$. A legal branch combines an immobile site of capacity H_r^{site}, local mobile workers $L_{fr,t}$ and headquarters services $H_{f,t}$ shared across its parent's sites, as in Kleinman [2025]; the spatial setting follows Desmet and Rossi-Hansberg [2014]. Its site composite is $\Phi_{fr,t} = (H_r^{site})^{1-\alpha_\Sigma} H_{f,t}^{\alpha_H} L_{fr,t}^{\alpha_L}$, with $\alpha_\Sigma = \alpha_H + \alpha_L < 1$.

Machines receive the share $1-\beta_m$ of after-royalty sales and the site's own inputs the share β_m.

Proposition 4.2 (Branch Accounts and Network Scope)**.** *At $w_t^L > 0$, an operating branch with positive headquarters, site capacity and producer receipt chooses a unique positive employment. The local wage bill is the share α_L of its pre-wage operating surplus. For a certified independent machine supplier on the unit-quote branch, require $(1-\beta_m)(1+\gamma_C)^{\beta_m/(1-\beta_m)} > 1$ so the improved design beats the competitively copyable predecessor. With gross sales $S_r^{sales} = h_t Q_{fr,t}$, the exhaustive operating account is*

$$\begin{array}{ll} \textit{public royalty} & \tau_r^G S_r^{sales} \\ \textit{local ordinary wages} & \alpha_L\beta_m(1-\tau_r^G)S_r^{sales} \\ \textit{branch contribution before HQ} & (1-\alpha_L)\beta_m(1-\tau_r^G)S_r^{sales} \\ \textit{independent patent profit} & \beta_m(1-\beta_m)(1-\tau_r^G)S_r^{sales} \\ \textit{real machine manufacturing} & (1-\beta_m)^2(1-\tau_r^G)S_r^{sales} \end{array} \tag{65}$$

For competitive public dirty machines, the patent-profit row is zero and the manufacturing row is $(1-\beta_m)p_{r,t}Q_{fr,t}$. After choosing local labor, branch contribution is $C_{fr,t}^F H_{f,t}^{\varpi_H}$, with $\varpi_H = \alpha_H/(1-\alpha_L)$. Let $B_{f,t}^F = \sum_{r\in\mathcal{N}_{f,t}} C_{fr,t}^F$ over the parent's network $\mathcal{N}_{f,t}$. With optimal headquarters the parent's operating value is

$$\Psi(B;w) = (1-\varpi_H)B(\varpi_H B/w)^{\varpi_H/(1-\varpi_H)} \tag{66}$$

and a new branch r adds

$$\Delta\Pi_{r,t}^F(\tau) = \Psi(B_{f,-r,t}^F + C_{fr,t}^F(\tau); w_t^L) - \Psi(B_{f,-r,t}^F; w_t^L) \tag{67}$$

At fixed wages and receipts, Ψ is strictly convex in $B > 0$, so an existing network raises the incremental operating value of the same branch (see Online Appendix I.1 and equation 130).

The account separates three claimants on the same sales. The government's royalty comes off the top, the patent owner's margin is fixed by the machine technology, and the parent keeps the branch contribution after paying headquarters and site bills. Because headquarters are shared, the parent's value is convex in its network: the same mine is worth more to a larger parent. The parent opens the branch when $\Delta\Pi^F$ covers setup, access and adoption costs and can be financed.

For growth, a competitive final producer combines certified and uncertified gold with a CES technology of elasticity $\varepsilon_M > 1$ and weight γ_M on certified gold. Income and the ordinary wage w^L are now endogenous, and transfers redistribute receipts without creating resources. This closure replaces the fixed North and South budgets of Section 4.1.

Theorem 4.3 (Relative Decline of Uncertified Income)**.** *Fix an active company network with positive finite company moments and positive available ordinary labor $\bar{L}^{prod}$. The market has a unique positive price ratio, price pair and ordinary wage. Along a certified-directed path suppose $A_C \to \infty$, while A_D remains in a compact positive interval; the*

company moments are bounded above, both sectors' mobile moments are bounded away from zero, $Z^{D,coer}$ is bounded, and $\bar{L}^{prod}$ remains in a compact positive interval. Then, with $\mathcal{D}_\Sigma = 1 + \beta_m(1-\alpha_\Sigma)(\varepsilon_M - 1) > 1$ and $\theta_\Sigma = \mathcal{D}_\Sigma^{-1} < 1$,

$$\begin{aligned} p_D &= O(A_C^{\beta_m \theta_\Sigma}), \\ p_C &\longrightarrow \gamma_M^{\varepsilon_M/(\varepsilon_M-1)}, \qquad w^L = \Theta(A_C), \\ p_D Q_D^{coer} &= O(A_C^{\theta_\Sigma}), \qquad p_D Q_D^{mobile} = O(A_C^{e_\Sigma}), \\ e_\Sigma &= \frac{1 - \alpha_\Sigma \beta_m(\varepsilon_M - 1)}{\mathcal{D}_\Sigma} < \theta_\Sigma < 1 \end{aligned} \tag{68}$$

Certified operating opportunities have order A_C. Any flow bounded by a fixed multiple of dirty sales plus a bounded dated claim, such as protection payments under the affordability ceiling of Section 5.1 or a one-period dirty research gain, is at most $D_0 + D_1 A_C^{\theta_\Sigma}$: it can grow in levels while falling relative to certified opportunities (see Online Appendix I.3).

As certified technology advances, certified gold becomes abundant and uncertified gold becomes the relatively scarce input. The certified price settles at a positive limit while the uncertified price can keep rising; in the long run it exceeds the certified price. A rising price does not keep uncertified income in step with the economy. Final producers substitute toward certified gold, $\varepsilon_M > 1$, so uncertified receipts grow at most on the order of $A_C^{\theta_\Sigma}$ while the wage grows with A_C. Absolute income at an uncontrolled mine can rise. What falls is its size relative to the certified economy, and that ratio is what Section 5 uses. Aggregate uncertified extraction can still decline as the mass of uncontrolled regions shrinks.

A newly successful certified vintage whose quality is proportional to A_C, with its licensed-market multiplier bounded above and away from zero, has a first operating flow of order A_C, while an old surviving vintage keeps its own quality (see Online Appendix I.4). The same scale reaches the public accounts of a legal mine.

Corollary 4.4 (Royalty Flow from a Legal Mine)**.** *Under Theorem 4.3, for a maintained-active legal branch, suppose its fixed royalty converges to a value strictly between zero and one, its site and adoption coefficients have positive limits, its own parent network and coefficients converge, and the aggregate moments and available labor converge. Then $T_{r,t}/A_{C,t} \to t_r > 0$, where $T_{r,t} = \tau_r^G h_t Q_{fr,t}$ is the royalty (see Online Appendix I.5).*

The royalty is gross of household support, public services and adoption costs, which Section 5.1 subtracts.

4.4 Research Rewards, Support and Path Dependence

Each sector has a unit measure of design lines and finitely many project types of masses $N_i > 0$, with $\sum_{i \in j} N_i \leq 1$ in each sector j. Projects receive distinct lines by a lottery independent of quality and effort, and a challenger targets a patent owned by someone else.

The effort that creates new certified patents also threatens existing ones: a type-i project succeeds with probability $h_i(s) = 1 - e^{-\nu_i^R s}$ at effort s, and a certified line is replaced with probability $\rho_{C,t} = \sum_{i \in C} N_i h_i(s_{i,t})$ (see Online Appendix, equation 113).

Let I^v_{t+1+n} indicate that the certified patent survives into production period $t+1+n$, π^v its licensed machine profit, $\Delta\mathcal{O}_{D,t+1}$ the one-interval operating gain on an uncertified prototype, and $\mathcal{C}_{i,t}$ the discounted incremental deployment costs: installation, access, collateral and later contractual payments, net of refunds. Each reward is the value with success minus the value with failure; research wages and laboratory costs enter separately. After diffusion an uncertified improvement is common knowledge, so later productive gains are the same with or without success and cancel, while contractual payments still due remain in $\mathcal{C}_{D,t}$. The date-t rewards conditional on success are

$$\begin{aligned} J_{C,t} &= \mathbb{E}_t\left[\sum_{n\geq 0} \delta_R^{n+1} I^v_{t+1+n} \pi^v_{t+1+n}\right] - \mathcal{C}_{C,t}, \\ J_{D,t} &= \delta_R \mathbb{E}_t[\Delta\mathcal{O}_{D,t+1}] - \mathcal{C}_{D,t} \end{aligned} \tag{69}$$

A certified patent keeps the quality of its own vintage, not later improvements by others. Its value is the unique solution of its survival recursion that satisfies a transversality condition, even when the frontier grows faster than $1/\delta_R$ (see Online Appendix I.6). Researchers choose effort by comparing the expected reward with the scientist wage and laboratory costs. One fixed mass S^{sci} of scientists serves research and verification, and at any finite rewards a competitive allocation with a nonnegative scientist wage exists (see Online Appendix I.7).

Let $0 \leq S^{\text{ver}} < S^{\text{sci}}$ be the scientists assigned to verifying current and outstanding vintages, and $s^*_C = (S^{\text{sci}} - S^{\text{ver}})/N_C$ the effort per certified project when all research is certified. Rewards are normalized by a common scale $c_t > 0$, $\widehat{J}_{i,t} = J_{i,t}/c_t$; the growth application sets $c_t = A_{C,t}$. Each type has a normalized laboratory cost $R_i^{\text{lab}}(s)$, convex, nondecreasing and differentiable with $R_i^{\text{lab}}(0) = 0$. A success payment has date-t value $c_t G_{i,t}$.

Proposition 4.5 (Minimum Research Support)**.** *Evaluate rewards along a public path consistent with full-certified research, holding rival current payments and the anticipated continuation fixed. Define*

$$\begin{aligned} \mathcal{R}^{\text{alt}}_t &= \max\left\{0, \max_{i \neq C}\left[h_i'(0)[\widehat{J}_{i,t} + G_{i,t}]_+ - R_i^{\text{lab}\prime}(0)\right]\right\}, \\ \Gamma_t &= \frac{R_C^{\text{lab}\prime}(s^*_C) + \mathcal{R}^{\text{alt}}_t}{h_C'(s^*_C)} - \widehat{J}_{C,t}, \qquad G^{\min}_{C,t} = [\Gamma_t]_+ \end{aligned} \tag{70}$$

A nonnegative additive success payment weakly supports full-certified research if and only if $G_{C,t} \geq G^{\min}_{C,t}$. Strict marginal dominance requires $G_{C,t} > \Gamma_t$ and $G_{C,t} \geq 0$. Thus $\Gamma_t < 0$ permits strictly private takeover, $\Gamma_t = 0$ is a research tie, and $\Gamma_t > 0$ requires positive support. A positive matching subsidy can implement this payment only when the underlying private prize is positive (see Online Appendix I.8).

In the two-type case of Theorem 4.6, with quadratic laboratory costs $R_i^{\text{lab}}(s) = r_i s^2/2$, effort $s^* = S^{\text{sci}} - S^{\text{ver}}$ and no payment to uncertified research, $h_D'(0) = \nu_D^R$ and $R_D^{\text{lab}\prime}(0) = 0$. The threshold then reads

$$G_{C,t}^{\min} = \left[\frac{r_C s^* + \nu_D^R[\widehat{J}_{D,t}]_+}{H_C} - \widehat{J}_{C,t}\right]_+, \qquad H_C = \nu_C^R e^{-\nu_C^R s^*} \tag{71}$$

The support equals the certified project's marginal laboratory cost plus the best rival's marginal return, divided by the certified marginal success probability, less the certified prize.

The rival set includes inactivity, valued at zero. More verification can lower the threshold while lowering certified effort, so a smaller grant need not mean more innovation (Online Appendix equation 143 gives the pooled–certified difference). Theorem 4.3 makes the uncertified reward vanish relative to A_C; private certified research also needs a certified reward that keeps pace with A_C. The coefficient $j_0 = \delta_R \beta_m (1+\gamma_C)\underline{\mathcal{M}} - \kappa_C$ bounds that reward from below, where $\underline{\mathcal{M}}$ bounds the licensed market of a patent from below and $\kappa_C A_C + K_C$ bounds its deployment costs (see Online Appendix I.9).

Theorem 4.6 (Temporary Support and Private Continuation). *1. Suppose throughout a forward-invariant domain of technologies, licensed networks, adoption states, verification workloads, claims and funding that unsubsidized values obey the reward bounds below, all dirty and other competing marginal returns have a vanishing upper envelope, and normalized laboratory marginal costs are uniformly bounded. Suppose the certified marginal surplus has a strictly positive limiting lower bound. Then a sufficiently large certified technology level supports full-certified research strictly at every subsequent date, provided the certified technology scale is nondecreasing.*

2. Consider two homogeneous research types with $N_C = N_D = 1$, fixed $S^{\text{ver}} < S^{\text{sci}}$, $s^* = S^{\text{sci}} - S^{\text{ver}}$, and $R_i^{\text{lab}}(s) = r_i s^2/2$, $r_i \geq 0$. Maintain the reward bounds

$$\widehat{J}_{C,t} \geq j_0 - \frac{K_C}{A_{C,t}}, \qquad [\widehat{J}_{D,t}]_+ \leq \frac{D_0}{A_{C,t}} + D_1 A_{C,t}^{\theta_\Sigma - 1} \tag{72}$$

uniformly on a forward-invariant market domain for $A_C \geq A_{\min} > 0$, with $K_C, D_0, D_1 \geq 0$. Define

$$\begin{aligned} m_\infty &= \nu_C^R e^{-\nu_C^R s^*} j_0 - r_C s^* > 0, \\ A^* &= \max\left\{A_{\min}, \frac{4H_C K_C}{m_\infty}, \frac{4\nu_D^R D_0}{m_\infty}, \left(\frac{4\nu_D^R D_1}{m_\infty}\right)^{1/(1-\theta_\Sigma)}\right\} \end{aligned} \tag{73}$$

At every $A_C \geq A^*$, zero support strictly sustains full-certified research.

3. In the deterministic cohort-balanced application,

$$A_{C,t+1} = G_A A_{C,t}, \qquad G_A = 1 + \gamma_C h_C(s^*) > 1 \tag{74}$$

Starting from $A_{C,0} \geq A_{\min}$, funded payments strictly satisfying Proposition 4.5 until

$$T^{\mathrm{wd}} = \max\left\{0, \left\lceil \frac{\log(A^*/A_{C,0})}{\log G_A} \right\rceil \right\}$$

reach the invariant private-green region. Payments can then cease permanently.

(see Online Appendix, Theorems I.10 and I.11)

Path dependence comes from inheritance. Each certified success raises the quality of the lines the next certified inventor inherits, and a patent on a better line is worth more. An uncertified improvement instead adds to a shared standard that no one owns; inherited dirty technology and outstanding claims enter the bounds D_0 and D_1 on competing opportunities. The cutoff A^* in (73) records this history: it rises with the certified fixed cost K_C and with the uncertified opportunity bounds, and falls with the certified margin m_∞. Temporary support changes the history: once certified technology passes A^*, certified research outbids every rival on its own, after at most T^{wd} funded rounds. The cutoff is sufficient: private takeover can also occur below it. Appendix D.2 combines the same reward bounds into a weakly smaller sufficient cutoff $A^\dagger \leq A^*$, which can replace A^* in the withdrawal-date formula.

Patent life enters the cutoff through the replacement hazard. The coefficient j_0 counts only a patent's first production flow. If the patent's licensed market stays above $\underline{\mathcal{M}}$ while it survives, its later rents raise the coefficient to

$$j_0^{\mathrm{surv}} = \frac{\delta_R \beta_m (1+\gamma_C)\underline{\mathcal{M}}}{1-\delta_R(1-\rho_C)} - \kappa_C, \qquad \rho_C = h_C(s^*) \tag{75}$$

(see Online Appendix K.19). The bounds of Theorem 4.6 then hold with j_0^{surv} in place of j_0, and the sufficient cutoff A^* weakly falls. The hazard ρ_C comes from the same research effort that creates the patent: creative destruction shortens the patent life that rewards certified research.

Withdrawal can fail. If the limiting certified marginal surplus is strictly negative, support is eventually needed even when uncertified returns vanish; if the licensed core, the branches that keep licensing the design, loses its market, the invariant region no longer applies (see Online Appendix I.12). With stochastic innovation, the cutoff is reached in finite time almost surely when technology never falls, each period carries a uniformly positive probability of a fixed proportional gain, and the programme stays funded until then. Section 5 starts on such a certified-directed path, privately sustained or supported. It uses two results from this section: uncertified income grows more slowly than A_C (Theorem 4.3), and the royalty of a legal mine grows in proportion to it (Corollary 4.4).

5 Building State Capacity after Certification

Certified growth does not starve the armed group. Theorem 4.3 allows the uncertified price, and the income of an uncontrolled mine, to rise in absolute terms. What changes is what the government gives up by leaving the region to the illegal group: a legal mine whose royalty grows with the certified frontier (Corollary 4.4). The government's alternative is the public value of legal production after services and household support, weighed against the protection payment collected by waiting and the cost and uncertain success of restoring control. Relative to work on state capacity and on taxation by armed groups [Besley and Persson, 2009, Henn et al., 2025], the return to governing here is driven by directed innovation in world markets: retaking a region becomes a real option whose payoff grows with the certified frontier. This section starts on a certified-directed path of Section 4.4. It derives the royalty a company accepts, the threshold at which intervention pays, the conditions under which every government eventually crosses it, and a bound on the damage done before it does.

5.1 Control, Legal Production and Public Accounts

Before control, the coerced Selva economy continues. Let $z^G_{r,t} \in \{0,1\}$ indicate control of region r and $i^G_{r,t} \in \{0,1\}$ an intervention attempt. The government decides before the current protection payment is made. Control takes effect at $t+1$ and ends new dirty production, its receipts and protection payments, though not existing gold stocks and claims (see Online Appendix, equation 123).

I take the protection payment $\widetilde{b}_{r,t}$ as given, possibly dependent on the public state, and leave the bargaining between the armed group and the government aside [Sánchez de la Sierra, 2020, Henn et al., 2025]. I impose one restriction, an affordability ceiling: the group cannot pay more than a fixed multiple of its dirty sales plus bounded dated claims. Under this ceiling Theorem 4.3 bounds protection income by $D_0 + D_1 A_C^{\theta_\Sigma}$, so a group that raises its payment to delay intervention cannot keep pace with a legal alternative that grows with A_C. That bound is all Theorem 5.2 uses.

A legal mine is a branch of a frontier parent at an immobile site, with mobile ordinary workers; the parent pays for headquarters, licensed machines and local adoption. Control, opening and local innovation are separate events. The public account is the royalty on legally eligible sales, less public services, any assigned research or adoption contribution and support for the original households. Support covers each household's shortfall below its entitlement and follows the household rather than local hiring (see Online Appendix, equations 124 and 125). After opening at date e_r, the branch catches up with the frontier through paid adoption. At local age a^{age} its standard is $A_{r,t} = \chi_r x_{r,a^{\text{age}}} A_{C,t}$, where χ_r is a

site coefficient and $x_{r,a^{\text{age}}} \in [0,1]$ its implementation level, and each purchased adoption service raises x by $\nu_r^{\text{ad}} > 0$. Financed without interruption, the branch graduates after

$$H_r^{\text{ad}} = \left\lceil \frac{1 - x_{r,0}}{\nu_r^{\text{ad}}} \right\rceil \tag{76}$$

periods, when local frontier research becomes eligible. Suppose the monetary scale grows by the factor G, $d^G = \delta_G G < 1$, fiscal flows are absolutely discounted-integrable, and the expected normalized mature flow is constant at r_∞^G. With $r_{a^{\text{age}}}^G$ the expected normalized fiscal flow at age a^{age}, net of subsistence support and public bills, the value of control is

$$\frac{U_{r,e_r}^{\text{ctrl}}}{B_{e_r}} = \sum_{a^{\text{age}}=0}^{H_r^{\text{ad}}-1} d^{G\,a^{\text{age}}} r_{a^{\text{age}}}^G + \frac{d^{G\,H_r^{\text{ad}}} r_\infty^G}{1 - d^G} \tag{77}$$

(see Online Appendix, equations 127–129). Catch-up postpones the mature fiscal flow by H_r^{ad} periods and charges the support bills of the catch-up years against it; these net flows enter the intervention decision of Section 5.3.

5.2 The Royalty a Company Accepts

The government announces a royalty before the company decides whether to open, and the company opens if its value with the branch is at least its value without it. Because the mine cannot move once opened, a later promise not to raise the royalty would not be credible; I assume the announced royalty is enforceable before entry. In the proposition, β_m is the share of after-royalty sales paid to the site's own inputs, α_L the local-labor exponent of the site, V_0^F the opening value at a zero royalty and C^F the opening cost.

Proposition 5.1 (Entry-Constrained Royalty)**.** *At fixed headquarters, fixed world prices and a stationary common-quote opportunity, write $n_L = 1/[\beta_m(1-\alpha_L)] > 1$. Suppose the company's opening value is exactly $V_0^F(1-\tau)^{n_L} - C^F$, with $V_0^F > 0$, $0 \le C^F \le V_0^F$, and all cash constraints leave the resulting interval available. With tax-independent public costs and opening at weak participation,*

$$\tau^{F,\max} = 1 - (C^F/V_0^F)^{1/n_L}, \qquad \tau^{G,*} = \min\{\beta_m(1-\alpha_L), \tau^{F,\max}\} \tag{78}$$

If $C^F > V_0^F$, no nonnegative royalty induces opening (see Online Appendix, equation 133).

Royalty revenue is the rate times a sales base that shrinks as the rate rises. At fixed headquarters it peaks at $1/n_L = \beta_m(1-\alpha_L)$. The government charges this revenue-maximizing rate unless the company would then stay out, in which case it charges the highest royalty compatible with entry, $\tau^{F,\max}$. When headquarters respond to the royalty, an interior stationary point of royalty revenue obeys

$$\tau_r^G = \frac{1}{n_L[1 + \varpi_H \omega_r/(1-\varpi_H)]}, \qquad \omega_r = \frac{C_{fr}^F}{B_f^F} \tag{79}$$

where ω_r is the branch's share of its parent's network (see Online Appendix, equation 134). The rate lies between $\beta_m(1-\alpha_H-\alpha_L)$ and $\beta_m(1-\alpha_L)$ and falls with the branch's share of its parent's network: a royalty on a large branch also lowers the parent's headquarters input, which amplifies the fall in the branch's sales. Because ω_r is evaluated at the chosen royalty, the condition is implicit, and the Online Appendix compares it with the boundary solutions on net control value. Holding world prices and wages fixed, a larger parent network raises a new branch's incremental value through shared headquarters (Proposition 4.2). It can therefore raise the optimized local fiscal value and expand the set of regions in which intervention pays (see Online Appendix, equations 130–134).

5.3 When Intervention Pays

Intervention is an investment with an option to wait [McDonald and Siegel, 1986, Pindyck, 2000]. Waiting preserves current protection income and the option to attempt later; an attempt forgoes current protection and pays a cost for a chance of legal value next period; after failure the option to attempt later remains. When every compared policy is funded, the government's value satisfies

$$V_{r,t} = \max\Bigg\{\widetilde{b}_{r,t} + \delta_G V_{r,t+1}, \max_{\tau\in\mathcal{T}^{\text{tax}}_{r,t+1}}\Big[-K^G_{r,t}(\tau) + \delta_G P_{r,t}(\tau)U^{\text{ctrl}}_{r,t+1}(\tau) + \delta_G(1-P_{r,t}(\tau))V_{r,t+1}\Big]\Bigg\} \tag{80}$$

Here $\widetilde{b}$ is the protection payment available by waiting, K^G the cost of an attempt, P its probability of success and U^{ctrl} the value after control under a feasible tax. When finance binds, waiting and a failed attempt leave different balances, and the state-dependent problem replaces (80) (see Online Appendix E.9). I normalize protection, attempt cost and control value by the monetary scale B_t, which equals $A_{C,t}$ in the homogeneous application, and write $d^G = \delta_G B_{t+1}/B_t$.

Theorem 5.2 (Intervention Threshold and Eventual Intervention)**.** *Suppose the compared policies are funded and the reduced Bellman equation applies.*

1. *In a stationary normalized economy with $0 \le d^G < 1$, given $P > 0$, constant $b \ge 0$, a nonempty compact feasible tax set and continuous $u(\tau), k(\tau)$, let*

$$A^{\text{int}} = \max_{\tau\in\mathcal{T}^{\text{tax}}}\{d^G P u(\tau) - k(\tau)\}, \qquad b^{\text{crit}} = \frac{1-d^G}{1-d^G+d^G P}A^{\text{int}} \tag{81}$$

Immediate intervention, repeated after failure, is strictly optimal exactly when $b < b^{\text{crit}}$; equality is indifference. If k is tax-independent, the maximizing tax is the fiscal optimum of the Online Appendix, equation 132. If normalized legal flow R^G_t/B_t is constant at r^G, then $u = r^G/(1-d^G)$.

2. *On a changing path with tax-independent K_t^G, P_t, assume bounded normalized primitives, ${d^G}_t \le \bar{d^G} < 1$ on a tail, and*

$$ {d^G}_t \to d^G < 1, \quad P_t \to P > 0, \quad b_t \to 0, \quad k_t \to k, \quad u_t \to u > 0, \qquad k < d^G P u $$

Then every remaining government of that type strictly attempts at every sufficiently late date on that funded path. The conclusion is uniform across types or public histories when the bounds, convergence and strict margin are uniform.

(see Online Appendix J.1; Appendix E.1 extends part 1 to tax-dependent success and paid catch-up)

The term $A^{\text{int}}/(1 - d^G + d^G P)$ is the value of attempting every period until success. The threshold b^{crit} is its per-period equivalent: the government acts when protection income is worth less per period than the intervention programme. With a tax-independent attempt cost and a constant normalized legal flow r^G at the optimal royalty, part 1 gives $u = r^G/(1 - d^G)$ and

$$ b^{\text{crit}} = \frac{d^G P r^G - (1 - d^G)k}{1 - d^G + d^G P} \tag{82} $$

With certain success the government intervenes when protection income is below $d^G r^G - (1 - d^G)k$: the discounted legal flow less the annuity value of the attempt cost.

Part 2 is where the argument closes like a pair of scissors. The macro blade is Section 4. Under the affordability ceiling of Section 5.1, certified-directed growth bounds the income of an uncontrolled mine by a multiple of $A_C^{\theta_\Sigma}$ with $\theta_\Sigma < 1$ (Theorem 4.3), so normalized protection income vanishes, $b_t \to 0$, even while it grows in absolute terms. The micro blade is the legal mine. Its royalty grows in proportion to A_C (Corollary 4.4), so the normalized value of control u stays positive when services, catch-up and household support leave a surplus. Neither blade cuts alone. A relative decline in illegal income does not make a legal mine valuable, and a valuable legal mine does not by itself guarantee intervention when protection income grows as fast. Together they push protection income below the threshold: if $k < d^G P u$, every government of that type eventually attempts. Funding is a separate requirement. A government with finite reserves and no replenishment can afford only finitely many attempts, so the result holds on the histories its funding covers (see Online Appendix J.3).

5.4 Scarcity Rents and Market Feedback

Governments decide against a common market path, and their decisions change it: successful controls alter production, labor demand, headquarters inputs and research returns. Reform can strengthen the regions that do not reform. Removing uncertified capacity makes the remaining uncertified gold scarcer, so the receipts of a mine still under the armed group can rise as other groups lose theirs. Theorem 5.2 needs neither complementarity among governments nor a fall in protection income in levels; it uses only protection income

relative to the certified scale (see Online Appendix J.4). For finitely many regional cohorts with a fixed parent map, a policy equilibrium that reconciles each government's choice with the market path it helps generate exists in a regular class (see Online Appendix E.11). The comparison model of the Online Appendix gives an explicit case in which removing dirty capacity raises a remaining producer's protection rent (see Online Appendix N.22).

5.5 Safe Sites and Cumulative Disturbance

Control is not the conservation outcome. Each critical site must reach its safe allocation of Assumption 1.6. In the zero-critical-disturbance application, control ends harmful extraction at the protected pit and the legal branch mines a separate noncritical deposit, with paid access and installation. Certification alone cannot make legal mining at the protected pit safe. Let $M^D_{T^{\text{att}}}$ be the scale-weighted mass of regions still ungoverned at date T^{att}.

Proposition 5.3 (Bound on Cumulative Disturbance)**.** *Suppose all still-ungoverned regions attempt every date from T^{att}, every such attempt is funded, conditional success probabilities are at least $P_{\min} > 0$, and successful control is absorbing. Put $q^{\text{fail}} = 1 - P_{\min}$.*

1. *A given region remains uncontrolled after n attempts with probability at most q^{fail^n}, and succeeds at a finite random time almost surely. Expected ungoverned mass is at most $q^{\text{fail}^n} M^D_{T^{\text{att}}}$. For a finite set of critical sites, if success permanently implements their respective safe allocations, all sites are safe after an almost-surely finite random date, without independence across sites.*

2. *Suppose disturbance per still-uncontrolled site at date $T^{\text{att}} + n$, on every history, is bounded by its date-T^{att} envelope times g_D^n, and let $D_{T^{\text{att}}}$ be the integral of those initial envelopes. If $q^{\text{fail}} g_D < 1$, expected dirty disturbance tends to zero and its cumulative tail is at most $D_{T^{\text{att}}}/(1 - q^{\text{fail}} g_D)$. Every positive tolerance for expected aggregate disturbance is crossed at a finite date.*

3. *Let $\mathcal{D}_{<T^{\text{att}}}$ be cumulative pre-T^{att} disturbance, and $\mathcal{D}^{\text{legal}}_{\geq T^{\text{att}}}$ the complete additional legal-disturbance stream, each measured in the same expected or exact sense. The irreversible-damage allowance is satisfied if*

$$\mathcal{D}_{<T^{\text{att}}} + \frac{D_{T^{\text{att}}}}{1 - q^{\text{fail}} g_D} + \mathcal{D}^{\text{legal}}_{\geq T^{\text{att}}} \leq \mathcal{B} \tag{83}$$

   For zero legal disturbance and uniformly bounded dirty intensity $\bar{d}_a$, the sufficient bound is $\sum_a \bar{d}_a M^D_{a,0}(T^{\text{att}} + 1/P_a) \leq \mathcal{B}$.

(see Online Appendix J.2)

Aggregate statements are in expectation; they hold pathwise under the exact aggregation law of the Online Appendix. The condition $q^{\text{fail}} g_D < 1$ requires the contraction in

uncontrolled mass to outpace any growth in damage at the sites that remain. With a continuum of critical sites, expected damage can vanish while some sites remain uncontrolled at every finite date. No later control or legal production reverses damage already done.

5.6 How the Illegal Trade Ends

Rules on buyers leave extraction in place when unaffected buyers and laundering set the armed group's payments (Section 2). On the partial-fraud branch of Section 3, certification leaves a positive fraction of fraudulent admissions. Neither removes the armed group from the mine. A government that retakes the region does. Certified-directed growth eventually makes that choice worthwhile: the value of a legal mine grows with the certified frontier while protection income does not keep pace (Theorem 5.2). Once every remaining government attempts and each attempt succeeds with probability at least $P_{\min}$, expected uncontrolled mass declines geometrically, and expected damage at the remaining sites tends to zero if $q^{\text{fail}} g_D < 1$ (Proposition 5.3). Because damage is irreversible, the transition protects the forest only if it begins early enough for (83) to hold.

6 Conclusion

Rules on buyers change demand, but their effect on extraction depends on the institutions between buyers and mines. An armed group that controls local trade can reallocate gold among buyers, retain it or launder it, so its income can fall while extraction does not. Households that mine to meet basic needs can even mine more.

A physically backed record of origin changes what can be sold, not only who buys. It moves the problem of fraud to the mine, where its extent depends on how inspection, collateral and disclosure are financed. Once origin is verifiable, the market for the same metal splits in two. The uncertified price falls when Northern demand is large enough, and the floor that laundering put under the armed group's receipts falls with it. Innovation responds to what inventors keep: certified patents last until displaced, while uncertified improvements diffuse after one period. Temporary support can set certified research on a path where it sustains itself.

The illegal trade ends when governments retake the regions where it operates, and certified growth gives them a reason to. It does not starve the armed group, whose protection payments can rise. But those payments grow more slowly than the royalty of a legal mine, which grows with the certified frontier. The two blades close: protection income falls below the intervention threshold, and governments that can fund repeated attempts eventually intervene. Control protects the forest only where legal production is itself safe, and only if it arrives while cumulative damage is still within what the forest can absorb.

The urgency extends beyond the modeled market. The Uppsala Conflict Data Program recorded 65 state-based armed conflicts in 2025, and gold has become a major source of income for the warring parties in Sudan [Davies et al., 2026, Soliman and Baldo, 2025].

Several links remain open. I do not derive the transition from the certification date to the growth path, which use different demand systems, nor compare certification with pooling along entire equilibrium paths; the one-interval benchmark of the Online Appendix compares their takeover cutoffs directly. Protection payments are taken as given, and bargaining between armed groups and governments would make them endogenous.

The design of custody and of the record is also open. Decentralized custody distributes control but creates more occasions to substitute physical gold; centralized custody concentrates that risk in one institution. A single ordering authority concentrates control, while multiple validators raise questions of coordination. Shared evidence can invite free riding, and additional ledger nodes do not themselves supply independent enforcement. Cryptography authenticates records of physical backing, while inspection, custody and enforceable claims determine whether the recorded origin can be trusted. A fistful of gold can pay for a war or help finance a state. Who holds it, and who can verify its origin, matters for both.

References

Ryan Abman. Rule of law and avoided deforestation from protected areas. *Ecological Economics*, 146:282–289, 2018.

Daron Acemoglu. Directed technical change. *Review of Economic Studies*, 69(4):781–809, 2002.

Daron Acemoglu. Why not a political coase theorem? social conflict, commitment, and politics. *Journal of Comparative Economics*, 31(4):620–652, 2003.

Daron Acemoglu. Politics and economics in weak and strong states. *Journal of Monetary Economics*, 52(7):1199–1226, 2005. doi: 10.1016/j.jmoneco.2005.05.001. URL https://doi.org/10.1016/j.jmoneco.2005.05.001.

Daron Acemoglu. Modeling inefficient institutions. In Richard Blundell, Whitney K. Newey, and Torsten Persson, editors, *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress*, volume 1, chapter 9, pages 341–380. Cambridge University Press, 2006. doi: 10.1017/CBO9781139052269.011.

Daron Acemoglu and James A. Robinson. Political losers as a barrier to economic development. *American Economic Review*, 90(2):126–130, 2000.

Daron Acemoglu and James A. Robinson. Economics versus politics: Pitfalls of policy advice. *Journal of Economic Perspectives*, 27(2):173–192, 2013. doi: 10.1257/jep.27.2.173. URL https://www.aeaweb.org/articles?id=10.1257/jep.27.2.173.

Daron Acemoglu, Pol Antràs, and Elhanan Helpman. Contracts and technology adoption. *American Economic Review*, 97(3):916–943, 2007.

Daron Acemoglu, Davide Ticchi, and Andrea Vindigni. Emergence and persistence of inefficient states. *Journal of the European Economic Association*, 9(2):177–208, 2011.

Daron Acemoglu, Philippe Aghion, Leonardo Bursztyn, and David Hemous. The environment and directed technical change. *American Economic Review*, 102(1): 131–166, 2012. doi: 10.1257/aer.102.1.131.

Daron Acemoglu, James A. Robinson, and Rafael J. Santos. The monopoly of violence: Evidence from colombia. *Journal of the European Economic Association*, 11(S1):5–44, 2013.

Daron Acemoglu, Camilo García-Jimeno, and James A. Robinson. State capacity and economic development: A network approach. *American Economic Review*, 105(8): 2364–2409, 2015. doi: 10.1257/aer.20140044. URL https://www.aeaweb.org/articles?id=10.1257/aer.20140044.

Daron Acemoglu, Ufuk Akcigit, Douglas Hanley, and William Kerr. Transition to clean technology. *Journal of Political Economy*, 124(1):52–104, 2016.

Rafael Araujo. When clouds go dry: An integrated model of deforestation, rainfall, and agriculture. *Working Paper*, 2024.

Rafael Araujo, Juliano Assunção, Marina Hirota, and José A. Scheinkman. Estimating the spatial amplification of damage caused by degradation in the amazon. *Proceedings of the National Academy of Sciences*, 120(46):e2312451120, 2023.

Rafael Araujo, Francisco Costa, and Marcelo Sant'Anna. Efficient conservation of the Brazilian Amazon: Estimates from a dynamic model. Working paper, version of 10 December 2024, 2024.

Juliano Assunção, Clarissa Gandour, and Romero Rocha. DETER-ing deforestation in the amazon: Environmental monitoring and law enforcement. *American Economic Journal: Applied Economics*, 15(2):125–156, 2023.

Juliano Assunção, Lars Peter Hansen, Todd Munson, and José A. Scheinkman. Carbon prices and forest preservation over space and time in the Brazilian Amazon. Technical Report 2023-86, Becker Friedman Institute, 2025. Revised 30 June 2025.

Lawrence M. Ausubel and Peter Cramton. The optimality of being efficient. Working paper, University of Maryland, June 1999. URL https://ausubel.com/auction-papers/99wp-optimality-of-being-efficient.pdf. Version dated 18 June 1999.

Clare Balboni, Aaron Berman, Robin Burgess, and Benjamin A. Olken. The economics of tropical deforestation. *Annual Review of Economics*, 15:723–754, 2023. doi: 10.1146/annurev-economics-090622-024705.

Mark Banchereau. Rebel group in Congo generates $300,000 monthly in seized mining area, UN says. Associated Press, 2024. URL https://abcnews.go.com/International/wireStory/rebel-group-congo-generates-300000-monthly-seized-mining-114377360.

Kathryn Baragwanath and Ella Bayi. Collective property rights reduce deforestation in the brazilian amazon. *Proceedings of the National Academy of Sciences*, 117(34): 20495–20502, 2020.

Björn Bartling, Roberto A. Weber, and Lan Yao. Do markets erode social responsibility? *Quarterly Journal of Economics*, 130(1):219–266, 2015. doi: 10.1093/qje/qju031.

Panle Jia Barwick, Shanjun Li, Liguo Lin, and Eric Yongchen Zou. From fog to smog: The value of pollution information. *American Economic Review*, 114(5):1338–1381, 2024. doi: 10.1257/aer.20200956.

Kaushik Basu and Pham Hoang Van. The economics of child labor. *American Economic Review*, 88(3):412–427, 1998. URL https://qed.econ.queensu.ca/pub/faculty/sumon/basu_childlabor.pdf.

Dirk Bergemann and Stephen Morris. Information design: A unified perspective. *Journal of Economic Literature*, 57(1):44–95, 2019. doi: 10.1257/jel.20181489.

Nicolas Berman, Mathieu Couttenier, Dominic Rohner, and Mathias Thoenig. This mine is mine! how minerals fuel conflicts in Africa. *American Economic Review*, 107(6): 1564–1610, 2017. doi: 10.1257/aer.20150774. URL https://www.aeaweb.org/articles?id=10.1257/aer.20150774.

Timothy Besley and Torsten Persson. The origins of state capacity: Property rights, taxation, and politics. *American Economic Review*, 99(4):1218–1244, 2009. doi: 10.1257/aer.99.4.1218. URL https://www.aeaweb.org/articles?id=10.1257/aer.99.4.1218.

Bloomberg News. Mining murders show Peru's struggles with illegal gold rush. Bloomberg News, republished by MINING.COM, May 2025. URL https://www.mining.com/web/mining-murders-show-perus-struggles-with-illegal-gold-rush/. SNMPE describes REINFO as a shield used by illegal miners attacking formal operations.

Shawn Blore. Taxes, fees, royalties and the formalisation of ASGM producers and exporters. Report, planetGOLD, 29 May 2024. Version 1.0, p. 6. URL https://www.planetgold.org/sites/default/files/Taxes_Fees_Royalties_and_Formalisation_ASGM_Producers_Exporters.pdf.

Leonardo Bonilla-Mejía and Iván Higuera-Mendieta. Protected areas under weak institutions: Evidence from colombia. *World Development*, 122:585–596, 2019.

Chris A Boulton, Timothy M Lenton, and Niklas Boers. Pronounced loss of amazon rainforest resilience since the early 2000s. *Nature Climate Change*, 12(3):271–278, 2022.

A. Lans Bovenberg and Lawrence H. Goulder. Environmental taxation and regulation. In Alan J. Auerbach and Martin Feldstein, editors, *Handbook of Public Economics*, volume 3, chapter 23, pages 1471–1545. Elsevier, 2002.

Avishay Braverman and Joseph E. Stiglitz. Sharecropping and the interlinking of agrarian markets. *American Economic Review*, 72(4):695–715, 1982. URL https://documents1.worldbank.org/curated/en/975871468344645509/pdf/REP229000Share00of0agrarian0markets.pdf.

Robin Burgess, Matthew Hansen, Benjamin A Olken, Peter Potapov, and Stefanie Sieber. The political economy of deforestation in the tropics. *The quarterly journal of economics*, 127(4):1707–1754, 2012.

Gilles Chemla. Downstream competition, foreclosure, and vertical integration. *Journal of Economics & Management Strategy*, 12(2):261–289, 2003.

Nicola Comincioli, Christopher R. Knittel, Elsa A. Olivetti, Ilenia G. Romani, and Sergio Vergalli. Critical minerals for a just energy transition. *Annual Review of Resource Economics*, 17:241–262, 2025. doi: 10.1146/annurev-resource-011224-050431.

Brian R. Copeland and M. Scott Taylor. Trade, growth, and the environment. *Journal of Economic Literature*, 42(1):7–71, 2004.

Partha Dasgupta and Debraj Ray. Inequality as a determinant of malnutrition and unemployment: Theory. *Economic Journal*, 96(384):1011–1034, 1986. doi: 10.2307/2233171. URL https://debrajray.com/wp-content/uploads/2017/12/DasguptaRay86.pdf.

Shawn Davies, Therése Pettersson, and Magnus Öberg. Organized violence 1989–2025, and violent political protests. *Journal of Peace Research*, 63(4):705–723, 2026. doi: 10.1093/jopres/xjag046.

Oeindrila Dube and Juan F. Vargas. Commodity price shocks and civil conflict: Evidence from Colombia. *Review of Economic Studies*, 80(4):1384–1421, 2013. doi: 10.1093/restud/rdt009. URL https://doi.org/10.1093/restud/rdt009.

Esther Duflo, Michael Greenstone, Rohini Pande, and Nicholas Ryan. Truth-telling by third-party auditors and the response of polluting firms: Experimental evidence from India. *Quarterly Journal of Economics*, 128(4):1499–1545, 2013. doi: 10.1093/qje/qjt024. URL https://doi.org/10.1093/qje/qjt024.

P. Dunia Kabunga and Sara Geenen. Work regimes and technology-driven transformations in DRCongo's artisanal and small-scale gold mines. *The Extractive Industries and Society*, page 101142, 2022. doi: 10.1016/j.exis.2022.101142. URL https://medialibrary.uantwerpen.be/files/1253738/c5e9627d-0c99-4e4b-a19f-cbc89a4eeb5b.pdf.

European Parliament and Council of the European Union. Regulation (EU) 2017/821 laying down supply chain due diligence obligations for union importers of tin, tantalum and tungsten, their ores, and gold originating from conflict-affected and high-risk areas. Official Journal of the European Union, L 130, 19 May 2017, May 2017. Adopted 17 May 2017; due diligence obligations apply from 1 January 2021.

Benjamin Faber, Benjamin Krause, and Raúl Sánchez de la Sierra. Artisanal mining, livelihoods, and child labor in the cobalt supply chain of the democratic republic of congo. Policy report, Center for Effective Global Action, University of California, Berkeley, May 2017. URL https://escholarship.org/uc/item/17m9g4wm.

Farid Farrokhi and Ahmad Lashkaripour. Can trade policy mitigate climate change? *Econometrica*, 93(5):1561–1599, 2025.

Farid Farrokhi, Elliot Kang, Heitor S. Pellegrina, and Sebastian Sotelo. Deforestation: A global and dynamic perspective. Working paper, December 2025 version, 2025.

Fabian Federl. On the trail of gold. *Pulitzer Center*, 24 November 2024. English translation of reporting for *DIE ZEIT*. URL https://pulitzercenter.org/stories/trail-gold.

Bernardo M. Flores, Encarni Montoya, Boris Sakschewski, Nathalia Nascimento, Arie Staal, Richard A. Betts, Carolina Levis, David M. Lapola, Adriane Esquível-Muelbert, Catarina Jakovac, et al. Critical transitions in the amazon forest system. *Nature*, 626 (7999):555–564, 2024.

GS1. *EPCIS and CBV Implementation Guideline*. GS1, 2.0 edition, 2023. URL https://ref.gs1.org/guidelines/epcis-cbv/2.0.0/. Accessed 8 September 2026.

Hanna Halaburda. Blockchain revolution without the blockchain? *Communications of the ACM*, 61(7):27–29, 2018.

Robert Heinkel, Alan Kraus, and Josef Zechner. The effect of green investment on corporate behavior. *Journal of Financial and Quantitative Analysis*, 36(4):431–449, 2001.

Soeren J. Henn, Christian Mastaki Mugaruka, Miguel Ortiz, Raúl Sánchez de la Sierra, and David Qihang Wu. Monopoly of taxation without a monopoly of violence: The weak state's trade-offs from taxation. *Review of Economic Studies*, 92(2):1126–1156, 2025. doi: 10.1093/restud/rdae036.

Jeffrey I. Herbst. *States and Power in Africa: Comparative Lessons in Authority and Control*. Princeton University Press, Princeton, NJ, 2000.

International Energy Agency. Global critical minerals outlook 2025. Technical report, International Energy Agency, 2025. URL https://www.iea.org/reports/global-critical-minerals-outlook-2025/executive-summary. Executive summary; accessed 8 September 2026.

Seema Jayachandran, Joost De Laat, Eric F Lambin, Charlotte Y Stanton, Robin Audy, and Nancy E Thomas. Cash for carbon: A randomized trial of payments for ecosystem services to reduce deforestation. *Science*, 357(6348):267–273, 2017.

Shizuo Kakutani. A generalization of Brouwer's fixed point theorem. *Duke Mathematical Journal*, 8(3):457–459, 1941. doi: 10.1215/S0012-7094-41-00838-4. URL https://www.math.ucdavis.edu/~deloera/TEACHING/READINGSEMINAR/PAPERS/kakutanioriginal.pdf.

Rachel E. Kranton and Deborah F. Minehart. A theory of buyer-seller networks. *American Economic Review*, 91(3):485–508, 2001. doi: 10.1257/aer.91.3.485.

Vijay Krishna and Motty Perry. Efficient mechanism design. Technical Report 133, Center for Rationality and Interactive Decision Theory, Hebrew University of Jerusalem, 1998. URL https://ratio.huji.ac.il/files/dp133.pdf. Version dated 14 April 1998.

David M Lapola, Patricia Pinho, Jos Barlow, Luiz EOC Aragão, Erika Berenguer, Rachel Carmenta, Hannah M Liddy, Hugo Seixas, Camila VJ Silva, Celso HL Silva-Junior, et al. The drivers and impacts of amazon forest degradation. *Science*, 379(6630): eabp8622, 2023.

Arik Levinson and M. Scott Taylor. Unmasking the pollution haven effect. *International Economic Review*, 49(1):223–254, 2008.

Simon Loertscher and Ellen V. Muir. Monopoly pricing, optimal randomization, and resale. *Journal of Political Economy*, 130(3):566–635, 2022. doi: 10.1086/717894. URL https://doi.org/10.1086/717894.

London Bullion Market Association. Responsible gold guidance. LBMA, London, 2012. Core of the LBMA Responsible Sourcing Programme; compliance required for Good Delivery List refiners.

London Bullion Market Association. About good delivery, 2026. URL https://www.lbma.org.uk/good-delivery/about-good-delivery. Accessed 8 September 2026.

London Metal Exchange. Trading, 2026. URL https://www.lme.com/Trading. Accessed 8 September 2026.

Elies Lucio. For a handful of gold. Working paper, 13 September 2026.

Mihai Manea. Intermediation and resale in networks. *Journal of Political Economy*, 126 (3):1250–1301, 2018. doi: 10.1086/697205.

Mihai Manea. Bottleneck links, essential intermediaries, and competing paths of diffusion in networks. *Theoretical Economics*, 16(3):1017–1053, 2021. doi: 10.3982/TE4385.

Robert McDonald and Daniel Siegel. The value of waiting to invest. *Quarterly Journal of Economics*, 101(4):707–728, 1986. doi: 10.2307/1884175.

Paul R. Milgrom, Douglass C. North, and Barry R. Weingast. The role of institutions in the revival of trade: the law merchant, private judges, and the champagne fairs. *Economics & Politics*, 2(1):1–23, 1990.

Luc Moreau and Paolo Missier. PROV-DM: The PROV data model. W3c recommendation, World Wide Web Consortium, April 2013. URL https://www.w3.org/TR/prov-dm/. Recommendation of 30 April 2013; accessed 8 September 2026.

Paramita Mukherjee and Vivekananda Mukherjee. Assessing the scope of gold refineries in india: Evidence from survey data. *Resources Policy*, 69:101810, 2020. doi: 10.1016/j.resourpol.2020.101810.

Roger B. Myerson. Optimal auction design. *Mathematics of Operations Research*, 6(1): 58–73, 1981. doi: 10.1287/moor.6.1.58.

OECD. *OECD Due Diligence Guidance for Responsible Supply Chains of Minerals from Conflict-Affected and High-Risk Areas: Third Edition*. OECD Publishing, Paris, 2016. doi: 10.1787/9789264252479-en. URL https://doi.org/10.1787/9789264252479-en.

OECD. Costs and value of due diligence in mineral supply chains. Oecd position paper, OECD Publishing, Paris, 2021a. URL https://doi.org/10.1787/4d432567-en.

OECD. Gold flows from Venezuela: Supporting due diligence on the production and trade of gold in Venezuela. OECD Publishing, Paris, 2021b. URL https://doi.org/10.1787/7a392f42-en.

OECD and UN. Conflict transformation and the role of responsible artisanal and small-scale mining: Supporting peace through supply chain due diligence, 2023. URL https://doi.org/10.1787/cdbd61d1-en.

R. Pacsi, S. Novoa, O. Yupanqui, M. Quispe, S. La Torre, H. Balbuena, B. Huamán, G. Valdivia, C. Castañeda, M. Soria, M. Finer, and A. Santana. Current situation of gold mining in the Peruvian Amazon. MAAP Report 233, Monitoring of the Andes Amazon Program, September 2025. URL https://www.maapprogram.org/gold-mining-peru-amazon/. Published 29 September 2025; observations through mid-2025.

Dominic P. Parker and Bryan Vadheim. Resource cursed or policy cursed? US regulation of conflict minerals and violence in the Congo. *Journal of the Association of Environmental and Resource Economists*, 4(1):1–49, 2017. doi: 10.1086/689865. URL https://www.journals.uchicago.edu/doi/10.1086/689865.

Cédric Perrin, Étienne Blanc, Olivier Cigolotti, and André Guiol. Le Plateau des Guyanes : affirmer la France comme un acteur régional clé. Rapport d'information 135 (2025–2026), Sénat, 19 November 2025. URL https://www.senat.fr/rap/r25-135/r25-1351.pdf.

Robert S. Pindyck. Irreversibilities and the timing of environmental policy. *Resource and Energy Economics*, 22(3):233–259, 2000.

Mathias Reynaert, Eduardo Souza-Rodrigues, and Arthur A van Benthem. The environmental impacts of protected area policy. *Regional Science and Urban Economics*, 107:103968, 2024.

Alvin E Roth. Repugnance as a constraint on markets. *Journal of Economic perspectives*, 21(3):37–58, 2007.

M. Ruiz-Vásquez, P. A. Arias, J. A. Martínez, and J. C. Espinoza. Effects of Amazon basin deforestation on regional atmospheric circulation and water vapor transport towards tropical South America. *Climate Dynamics*, 54:4169–4189, 2020. doi: 10.1007/s00382-020-05223-4.

Carla Samon Ros. Most of Peru's gold now comes from booming illegal mining sector. Bloomberg News, republished by MINING.COM, 2 October 2026, October 2026. URL https://www.mining.com/web/most-of-perus-gold-now-comes-from-booming-illegal-mining-sector/. Reports IPE estimates: illegally sourced gold was 54% of Peru's gold export volume, January–June 2026.

Raúl Sánchez de la Sierra. On the origins of the state: Stationary bandits and taxation in eastern congo. *Journal of Political Economy*, 128(1):32–74, 2020. doi: 10.1086/703989.

Leonardo A. Saravia, Santiago R. Doyle, and Ben Bond-Lamberty. Power laws and critical fragmentation in global forests. *Scientific Reports*, 8:17766, 2018. doi: 10.1038/s41598-018-36120-w.

Justin Scheck. Quienes certifican que el oro es limpio rara vez visitan las minas. *The New York Times*, Spanish publication by *Infobae*, 19 October 2026. URL https://www.infobae.com/america/the-new-york-times/2026/10/19/quienes-certifican-que-el-oro-es-limpio-rara-vez-visitan-las-minas/.

Joseph S. Shapiro. The environmental bias of trade policy. *Quarterly Journal of Economics*, 136(2):831–886, 2021.

Ahmed Soliman and Suliman Baldo. Gold and the war in Sudan: How regional solutions can support an end to conflict. Research paper, Chatham House, London, March 2025. URL https://www.chathamhouse.org/2025/03/gold-and-war-sudan.

Jean Tirole. Hierarchies and bureaucracies: On the role of collusion in organizations. *Journal of Law, Economics, and Organization*, 2(2):181–214, 1986. doi: 10.1093/oxfordjournals.jleo.a036907.

Robert M. Townsend. Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21(2):265–293, 1979. doi: 10.1016/0022-0531(79)90031-0. URL https://www.robertmtownsend.net/content/optimal-contracts-and-competitive-markets-costly-state-verification.

United Nations Environment Programme. Global mercury assessment 2018: Key findings. Technical report, United Nations Environment Programme, 2019. URL https://wedocs.unep.org/handle/20.500.11822/29830. Global anthropogenic atmospheric emissions inventory for 2015.

United Nations Group of Experts on the Democratic Republic of the Congo. Letter dated 27 December 2024 from the Group of Experts on the Democratic Republic of the Congo addressed to the President of the Security Council. United Nations Security Council, 2024.

U.S. Securities and Exchange Commission. Conflict minerals. Final Rule, Release 34-67716, U.S. Securities and Exchange Commission, August 2012. Adopted 22 August 2012. Economic analysis estimates initial compliance costs of $3–4 billion and ongoing costs of $207–609 million per year, for about 6,000 issuers.

Vlado Vivoda, Ron Matthews, and Neil McGregor. A critical minerals perspective on the emergence of geopolitical trade blocs. *Resources Policy*, 89:104587, 2024. doi: 10.1016/j.resourpol.2023.104587. Tables 1–2: refining shares far exceed reserve

shares (China 87% REE refining vs 33.9% reserves; 76% cobalt refining vs 1.6% reserves; DR Congo 67% cobalt mining).

World Gold Council. Gold market primer: Market size and structure, 2026. URL https://www.gold.org/goldhub/research/market-primer/gold-market-primer-market-size-and-structure. 18 August; accessed 8 September 2026.

Philippe Aghion and Peter Howitt. A model of growth through creative destruction. *Econometrica*, 60(2):323–351, 1992. https://dash.harvard.edu/entities/publication/73120378-cb4d-6bd4-e053-0100007fdf3b.

Benny Kleinman. Wage inequality and the spatial expansion of firms. Working paper, September 2025. https://binyamink.github.io/bennykleinman/bk_JMP.pdf.

Klaus Desmet and Esteban Rossi-Hansberg. Spatial development. *American Economic Review*, 104(4):1211–1243, 2014. https://rossihansberg.economics.uchicago.edu/SD.pdf.

Daron Acemoglu, Philippe Aghion, and David Hémous. The environment and directed technical change in a North–South model. *Oxford Review of Economic Policy*, 30(3): 513–530, 2014. doi: 10.1093/oxrep/gru031.

David Hémous. The dynamic impact of unilateral environmental policies. *Journal of International Economics*, 103:80–95, 2016. doi: 10.1016/j.jinteco.2016.09.001.

Jan Witajewski-Baltvilks and Carolyn Fischer. Green innovation and economic growth in a North–South model. *Environmental and Resource Economics*, 85:615–648, 2023. doi: 10.1007/s10640-023-00778-2.

Notation Tables

The table records the economic variables, sets and functions used in the main text and the paper's essential proofs. Subscripts identify households, buyers, sites, project types, dates or vintages as stated in each model; bound summation indices are local. Certified and uncertified sectors are indexed by C, D, and pooling and certification regimes by P, C. Descriptive qualifiers distinguish objects that previously shared a symbol. The Online Appendix retains its original notation: the final column gives the original names when a paper symbol has changed.

The literature supplies conventions for economic roles, not a universal choice of letters. Technology A, prices p, wages w, CES elasticities ε and discounted value functions follow standard usage. Here the machine exponent $1 - \beta_m$ corresponds to α in Acemoglu et al. [2012]; β_m is the own-input share, not that paper's machine exponent. Research success h_i and replacement ρ_C are one-period probabilities; ν_i^R is research productivity. The private-information, retention, certification and fiscal objects below are the paper's explicitly defined adaptations.

Table 1: Variables, Functions and Sets

Symbol	Definition and maintained domain	Location; original alias
Regional Production and Household Choices		
$\mathcal{G} = (\mathcal{V}, \mathcal{L}), \mathcal{J}, g$	Directed usable trade network, its vertices and links, regional sites, and controlled gate.	Section 1
$\mathcal{V}_J, \mathcal{V}^{\text{act}}$	Finite potential village set and selected active village network.	Section 1; active A
$u_v, \bar{U}_v$	Household utility and utility outside the gate's active network.	Section 1
C_v, c_v	Consumption and positive subsistence requirement; $C_v > c_v$. Unsubscripted C, c are the one-household benchmark.	Section 1
r_v, η_v, β_v	Positive leisure and positive utility weights on leisure and retained balances.	Section 1
a_v, b_v, b_{0v}	Spendable opening wealth, new retained cash and positive savings unavailable for current spending.	Section 1
α	Imported share of the Leontief consumption basket; $0 < \alpha < 1$.	Section 1
l_v^x, l_v^m, g_v	Local-good labor, mining labor and current imported mining inputs; nonnegative.	Section 1
F_v, G_v, K_v, A_D	Local-good and mineral technologies, inherited equipment and uncertified technology.	Sections 1, 4

Continued on the next page

Variables, Functions and Sets (continued)

Symbol	Definition and maintained domain	Location; original alias
θ_B, θ_E	Procurement quote and outside gross physical-gold quote per standardized unit.	Sections 1–2
c_v^z, c_v^g	Positive delivered resource costs of necessities and mining inputs.	Section 1
μ_z, μ_g	Common necessity and input margins; $\mu_z \geq 0$, while μ_g may be negative subject to positive delivered price.	Section 1
$p_v^z, p_v^g, p_v, \bar{p}_v$	Necessity, mining-input, local-good and composite consumption prices.	Section 1
A_m, A_x, w	Benchmark mining and local-good productivity and mining earning power $w = \theta_B A_m$.	Section 1; mining A
B^{sub}, d^x	Imported-basket cost αp^z and local-complement time $(1-\alpha)/A_x$.	Section 1; B, d
R^{time}, k_1, l	Benchmark residual time $1 - d^x a/B^{\text{sub}}$, coefficient $1 + d^x w/B^{\text{sub}}$, and mining labor.	Section 1; R
t_{vw}, δ_{vw}	Local shipments and iceberg factors; delivered shipment is t_{vw}/δ_{vw}, $\delta_{vw} \geq 1$.	Section 1
$e_v, \bar{e}_v, \Delta_v, \Psi_v$	Coercive effort, effort cap, outside-option reduction and real coercion cost.	Section 1
q, Z, I	Regional mineral stock/output, necessity imports and purchased mining-input total.	Sections 1–2
$z, \mathcal{Z}, \mathcal{Q}$	Full regional implementation, its feasible equilibrium graph and attainable operating quantities.	Sections 1–2
$T(z), C^{\text{impl}}(z)$	Transport/coverage bill and total regional implementation cost net of import-margin receipts.	Sections 1–2; $C(z)$
$\mathcal{C}(q), \Gamma^{\text{impl}}(q)$	Attained least implementation cost and its minimizing implementation correspondence.	Sections 1–2; $\Gamma(q)$
$W(q), \Pi$	Downstream stock value and gate profit, stock value minus implementation cost.	Sections 1–2
$\mathcal{E}, \mathcal{A}_r^{\text{safe}}$	Designated critical sites and their safe physical-allocation sets.	Sections 1, 5
$d_A, \bar{v}$	Imported-subsistence resource deficit of an active network and uniform stock-value bound per physical unit.	Essential Section 1 proofs
Selling Institution and Global Regulation		
$\bar{q}, Q, r_q, n$	Package size, complete-package count, fractional remainder, and offered package capacity.	Section 2

Continued on the next page

Variables, Functions and Sets (continued)

Symbol	Definition and maintained domain	Location; original alias
$\mathcal{I}, N, \bar{k}_i$	Distinct buyer owners, their number, and each buyer's public package capacity.	Section 2
$\omega_i, \omega, \nu_\omega$	Private buyer type, type profile and fixed joint type law; finite expected positive marginal values.	Section 2; law ν
$\mathcal{R}, \mathcal{U}, \tau^{\text{pol}}$	Regulated and unaffected buyer sets; nonnegative final-holder compliance charge per physical unit.	Section 2; τ
J_i^0, b_i^k, v_i, k_i	Operating value, marginal package value, post-compliance total buyer value and integer package allocation.	Section 2
$H(y), \ell, s, d^{\text{disp}}$	Gross retention value at retained stock y; laundering, storage and disposal quantities.	Section 2; disposal d
$V(\ell), S^{\text{stor}}(s)$	Laundering resource cost and net storage continuation value.	Section 2; storage $S(s)$
$S_n, S_{n,-i}$	Maximum total welfare with offered capacity n, with all buyers or with buyer i excluded.	Section 2
p_i, R_n, W_n	Whole-buyer pivot payment, seller cash plus true retained value, and its expectation over ν_ω.	Section 2
$\widehat{H}, T_0^{\text{net}}, y^{\text{final}}$	Announced retention schedule, cumulative net seller cash, and final retained stock in the complete game.	Section 2
$\Pi_q(\widehat{H}; H)$	Complete-game seller continuation payoff under announced and genuine retention schedules.	Section 2
$y_0, \rho_j(y), \rho(q)$	Stock outside offered capacity $q - n\bar{q}$, marginal retention orders and last one-package reserve.	Section 2; base a
$z_{-i,[j]}, c_{iu}$	Ranked rival orders after removing a whole buyer and the resulting critical payment slots.	Section 2
$m^{\text{sold}}, y, D_i(y)$	Number of packages sold, retained physical stock, and value of retaining buyer i's winning block.	Section 2; sold m
$p^R, L(q, p^R, \omega)$	Scalar regulated-demand index $\theta_E - \tau^{\text{pol}}$ and selected laundering quantity.	Section 2
$e_{\mathcal{U}}, e_{\mathcal{U},H}$	Minimum exclusion-book unaffected-demand floors, without and with retention orders.	Section 2
$k^{-i}, m_{\mathcal{R}}^{\text{sold}}, (m^{\text{sold}})_{\mathcal{R}}^{-i}$	Buyer-exclusion allocation and regulated package counts in actual and exclusion allocations.	Section 2; regulated m
$\chi_n, \bar{\chi}_n, \alpha_R$	Statewise and expected payment exposure; integrated transmission relative to offered capacity.	Section 2
$\Xi, \mathcal{X}, \Xi_0, \Delta_0(\Xi)$	Implementation/capacity choice, feasible choice set, baseline optimum and its initial profit gap.	Section 2

Continued on the next page

Variables, Functions and Sets (continued)

Symbol	Definition and maintained domain	Location; original alias
$\mathcal{X}^{\mathrm{eq}}, \kappa(t), \delta^{\mathrm{gap}}, \bar{Q}, T^{\mathrm{pol}}$	Equal-exposure comparison neighborhood, common exposure, uniform outside profit gap, package-count bound and policy horizon.	Section 2; B, δ
$\mathcal{H}, \boldsymbol{g}_q, \Lambda, \gamma$	Regional profit Hessian, output gradient, positive regional response factor and marginal payment exposure.	Section 2; gradient a
V_U, V_R, F_U, f_R	Independent unaffected/regulated unit-demand values, unaffected CDF and regulated density. Continuity at the reserve is used for the derivative.	Section 2; values U, R
$\underline{W}_n, \underline{W}, \underline{\Pi}$	Unaffected-only expected stock-value floors and optimized remaining-outlet profit floor.	Section 2
$c^{\mathrm{act}}_{\min}, q_0, q_\tau$	Uniform positive active implementation-cost floor, baseline output and policy-optimal output.	Section 2; cost floor d
$\boldsymbol{b}_i, U_i^M, U_i^V, c_i^{\mathrm{IR}}, x_i^{\mathrm{ord}}$	Buyer marginal-value vector; interim utility under an admissible mechanism and pivot mechanism, zero-type participation rent, and marginal-package allocation indicators.	Essential Section 2 proofs
Physical Certification and Investigation		
$c^{\mathrm{cl}}, v(c^{\mathrm{cl}}), \psi(c^{\mathrm{cl}}), \mu^{\mathrm{mass}}$	Live claim, recorded mass, identified backing and physical recoverable-content measure.	Section 3; claim c, measure m
$I^{\mathrm{cl}}, O^{\mathrm{cl}}, m_{\mathrm{loss}}, S_t, A_t, X_t$	Processing inputs/outputs, processing loss, live claims, cumulative admitted mass and cumulative loss/redemption mass.	Section 3
$\mathrm{Anc}, \mathcal{O}_t^{\mathrm{rev}}, \mathcal{O}_t^{\mathrm{susp}}, C_t$	Recorded origins, revoked/suspended origin admissions and eligible live claims.	Section 3
$L_{E,j}, L_{L,j}, L_{U,j}, P_{-j}^{\mathrm{disc}}$	Buyer's losses following early, late or no discovery, and other-channel discovery probability.	Section 3
$A_j^{\mathrm{disc}}, \pi_j, e_j, s_j, c_j$	Reporting advantage, fraud belief, investigation effort, conclusive-evidence probability and investigation cost.	Section 3; advantage A_j
$h, p_D, c^{\mathrm{forg}}, \bar{c}^{\mathrm{forg}}, \Gamma^{\mathrm{adm}}$	Clean procurement quote, committed dirty outside receipt, forgery cost/bound and pre-forgery substitution spread $h - p_D$.	Section 3; spread Γ
$\Omega, a^{\mathrm{div}}, d(a^{\mathrm{div}})$	Fixed committed market state, joint diversion action and diverted physical mass.	Section 3; action a
F_C, k_f, W_D, G_Ω	Net certified receipts, extra physical/concealment cost, dirty continuation value and gross diversion gain.	Section 3

Continued on the next page

Variables, Functions and Sets (continued)

Symbol	Definition and maintained domain	Location; original alias
$\mathcal{S}^{\mathrm{app}}, \mathcal{J}^{\mathrm{rep}}, \mathcal{T}^{\mathrm{sil}}$	Original approval coalition, purchasable evidence holders and selected silence set.	Section 3; sets S, J
$a_i(\mathcal{T}^{\mathrm{sil}}), \sigma_j(\mathcal{T}^{\mathrm{sil}}), \lambda_i$	Collectible loss, silence reservation compensation and delivered fraction of a gross side payment.	Section 3
$\mathcal{B}^{\mathrm{adv}}(\mathcal{T}^{\mathrm{sil}}), F_0^{\mathrm{adv}}, K_J^F$	Advance bribe/silence bill, available pre-settlement finance and minimum affordable capture cost (possibly $+\infty$).	Section 3; advance $\mathcal{B}$
$r_0, r_j, P_n^{\mathrm{disc}}$	Baseline/channel effective discovery probabilities and joint discovery probability under conditional independence.	Section 3
$B_0, b_i, D, F_{\mathrm{adv}}$	Organizer deposit, participant bonds, total collectible security and separate liquid bribe finance.	Section 3
$\phi, \phi_0, \Delta\phi$	Currency service fee, baseline service fee and additional scrutiny service cost per slot mass.	Section 3
$\kappa, L_E, L_L, r^{\mathrm{late}}$	Quadratic effort-cost coefficient, early/late losses and unsuppressible late-discovery probability.	Section 3; late r
$A^{\mathrm{disc}}, \alpha^{\mathrm{scr}}, \widetilde{\Gamma}^{\mathrm{adm}}$	Reporting advantage $r^{\mathrm{late}}L_L - L_E$, effort response A^{disc}/κ and substitution spread after expected baseline sanctions.	Section 3; $A, a, \widetilde{\Gamma}$
$\mathcal{L}^{\mathrm{buy}}(\pi, e), P^{\mathrm{disc}}(e)$, $\pi^*, e^*, P^{\mathrm{disc},*}$	Expected buyer cost, effective discovery, and interior equilibrium fraud, effort and discovery.	Section 3; cost R
$Q^{\mathrm{adm}}, U, R_U, R_A$	Admitted cohort mass, illegal admitted mass and illegal/admissible mass withdrawn later.	Section 3
$\iota^{\mathrm{in}}, \iota^{\mathrm{live}}, V_k, U_k, \mathcal{O}^{\mathrm{ill}}$	Admission/live contamination, surviving origin mass, its illegal component and true origins with illegal extraction.	Section 3; origin set $\mathcal{I}$
$w_k^{\mathrm{in}}, w_k^{\mathrm{live}}, \mathcal{C}_0, \mathcal{C}_1$	Admission/live origin weights and baseline/scrutiny institutional resource costs.	Section 3
World Prices, Production and Directed Innovation		
$\nu_{NS}, \phi_{NS}, \varepsilon_{NS}, \omega_{NS}$	North expenditure share $[0,1)$, ad valorem certified charge $[0,1)$, South CES elasticity > 1 and certified taste weight > 0.	Section 4; share ν
$p_C, p_D, p_0, \zeta, \bar{\zeta}$	Certified, uncertified and pooled output prices, price ratio p_C/p_D and supply-coefficient crossing ratio.	Section 4; ratio z
$m_\beta, q_j(\zeta), F, F_0, \bar{\nu}_{NS}$	Supply exponent $1/\beta_m$, sector supply coefficients, expenditure-weighted supply coefficient, pooled coefficient and segmentation threshold.	Section 4; exponent m

Continued on the next page

Variables, Functions and Sets (continued)

Symbol	Definition and maintained domain	Location; original alias
$s_0, E_C^C, E_C^P, M_C, \varrho^p$	Pooled clean spending share, certified producer net spending by regime, first-flow multiplier and dirty-price multiplier p_D/p_0.	Section 4; price factor d
$\beta_m, \alpha_H, \alpha_L, \alpha_\Sigma$	Own-input share $(0,1)$, HQ/local-labor exponents and their sum < 1. Machine share is $1-\beta_m$.	Section 4
$H_r^{site}, H_{f,t}, L_{fr,t}, \Phi_{fr,t}$	Immobile site capacity, shared HQ services, ordinary employment and site-input composite.	Section 4
$w_t^L, h_t, p_{r,t}^C, \tau_r^G$	Ordinary wage, certified gross quote, after-royalty producer receipt and public royalty rate.	Sections 4–5
$S_r^{sales}, C_{fr,t}^F, \mathcal{N}_{f,t}, B_{f,t}^F$	Branch sales, optimized branch coefficient, parent network and sum of its branch coefficients.	Section 4
$\varpi_H, \Psi(B;w), \Delta\Pi^F$	Reduced HQ exponent, optimized parent operating value and incremental value of a new branch.	Section 4
$\varepsilon_M, \gamma_M, \bar{L}^{prod}, Z^{D,coer}$	Growth-CES elasticity/weight, available ordinary labor and installed coerced-supply coefficient.	Section 4
$\mathcal{D}_\Sigma, \theta_\Sigma, e_\Sigma$	Positive growth denominator, its reciprocal < 1, and mobile-dirty receipt-growth exponent.	Section 4
$D_0, D_1, T_{r,t}, t_r$	Constant/sublinear dirty-income bounds, gross royalty flow and its positive limiting coefficient relative to A_C.	Sections 4–5
$N_i, s_{i,t}, \nu_i^R, h_i(s), \rho_{C,t}$	Project-type mass, effort, research productivity, one-period success probability and patent replacement probability.	Section 4
$\gamma_C, G_A, A^*, A_{\min}, T^{\text{wd}}$	Certified quality step, deterministic technology growth factor, sufficient private-continuation cutoff, domain minimum and funded withdrawal date.	Section 4
$I_{t+1+n}^v, \pi^v, \delta_R$	Patent survival indicator, vintage machine profit and research discount factor.	Section 4
$\Delta\mathcal{O}_D, \mathcal{C}_{i,t}, J_{i,t}, \widehat{J}_{i,t}, c_t$	Prototype operating gain, incremental deployment cost, complete/normalized success reward and positive monetary normalization.	Section 4
$S^{\text{sci}}, S^{\text{ver}}, s_C^*$	Total scientist mass, verification allocation and per-project full-certified effort.	Section 4; S, S_V
$R_i^{\text{lab}}(s), r_i, G_{i,t}$	Normalized laboratory-cost function, quadratic coefficient and normalized additive success payment.	Section 4; R_i
$\mathcal{R}_t^{\text{alt}}, \Gamma_t, G_{C,t}^{\min}$	Best rival/inactivity marginal return, untruncated support threshold and minimum nonnegative certified payment.	Section 4; rival $\mathcal{B}_t$

Continued on the next page

Variables, Functions and Sets (continued)

Symbol	Definition and maintained domain	Location; original alias
$H_C, j_0, \kappa_C, K_C, m_\infty$	Certified marginal success at full effort, reward lower coefficient, proportional/fixed deployment-cost bounds and positive limiting marginal surplus.	Section 4
$\underline{\mathcal{M}}, j_0^{\text{surv}}$	Lower surviving licensed-market coefficient and survival-refined reward coefficient.	Section 4
State Capacity, Fiscal Value and Conservation		
$z_{r,t}^G, i_{r,t}^G, \widetilde{b}_{r,t}$	Binary control and intervention-attempt indicators and protection payment obtainable by waiting.	Section 5
$e_r, a^{\text{age}}, x_{r,a^{\text{age}}}, \chi_r, \nu_r^{ad}, H_r^{ad}$	Opening date, local age, implementation fraction, site coefficient, paid adoption increment and graduation age.	Section 5
B_t, G, δ_G, d^G	Monetary normalization, its growth factor, government discount factor and normalized discount $\delta_G B_{t+1}/B_t < 1$.	Section 5; discount d
$r_{a^{\text{age}}}^G, r_\infty^G, R_t^G, U^{ctrl}, u$	Net normalized fiscal flow by age, mature flow, actual fiscal flow, control value and normalized control value.	Section 5
$V_0^F, C^F, n_L, \tau^{F,\max}, \tau^{G,*}$	Zero-royalty private opening value, opening cost, receipt elasticity and entry-constrained and chosen royalty rates.	Section 5
$\omega_r, \mathcal{T}_{r,t}^{\text{tax}}, K^G, P, V_{r,t}$	Branch share in its parent, feasible tax set, attempt cost, success probability and uncontrolled-government value.	Section 5
$b, k, u, d_t^G, A^{int}, b^{crit}$	Normalized protection, attempt cost, control value and discount, repeated-attempt numerator and protection threshold.	Section 5
$T^{\text{att}}, P_{\min}, q^{\text{fail}}, M_{T^{\text{att}}}^D, g_D, D_{T^{\text{att}}}$	First all-attempt date, minimum conditional success, failure bound $1 - P_{\min}$, remaining regional mass, dirty-disturbance growth bound and initial disturbance envelope.	Section 5; failure q
$\mathcal{D}_{<T}, \mathcal{D}_{\geq T}^{legal}, \mathcal{B}, \bar{d}_a$	Cumulative early damage, full later legal damage, irreversible-damage allowance and bounded dirty intensity by type.	Section 5
Additional Notation in the Paper Appendix		
$c^{\text{norm}}, s_x, R_i^{\text{lab}^x}, \mathcal{R}_x^{\text{alt}}, \Lambda_x, \Gamma_x$	Common normalization, effort, laboratory cost, rival return, break-even certified reward and support threshold by regime.	Preservation; c, R_i^x, B_x
$\Delta O_C, \Delta \mathcal{C}_C, m_P$	Complete operating-value and contractual-cost changes; strictly positive private pooled margin.	Preservation

Continued on the next page

Variables, Functions and Sets (continued)

Symbol	Definition and maintained domain	Location; original alias
$\zeta_\Sigma, \mathfrak{a}, \mathfrak{b}, f(A), \varrho, A^\dagger$	Exponent $1-\theta_\Sigma$, cutoff-bound coefficients, declining cost envelope, unique root and refined sufficient cutoff.	Withdrawal; exponent q
$z_r, B_0^{\text{prot}}, B_1^{\text{prot}}$	Fixed coerced-site supply coefficient and normalized protection-income bound coefficients.	Mine-receipt comparison
$\mathcal{T}^{\text{tax}}, k(\tau), P(\tau), u(\tau), r_{a^{\text{age}}}^G(\tau)$	Feasible tax set, tax-dependent funded attempt cost, success probability, control value and net public flows.	Intervention appendix

A Proofs: Selva: Production behind an Armed Chokepoint

These proofs establish the main results of Section 1 under their stated hypotheses. The regional reduction is conditional on attained implementation fibers and an attained profit maximum; it does not infer existence of every linked equilibrium from household concavity.

A.1 Household Incentives and Regional Resources

A.1.1 The Subsistence Labor Response

Proof of Proposition 1.3. On the maintained interior branch substitute $C = (a+wl)/B^{\text{sub}}$ and $r = R^{\text{time}} - k_1 l$ into utility. Its derivative in l is

$$\frac{w}{a + wl - B^{\text{sub}}c} - \frac{\eta k_1}{R^{\text{time}} - k_1 l}.$$

Both denominators are positive on this branch. The second derivative is strictly negative, so the first-order condition is necessary and sufficient. Cross multiplication gives $(1+\eta)wk_1 l = wR^{\text{time}} + \eta k_1(B^{\text{sub}}c - a)$, proving (5). Differentiation at fixed $a, B^{\text{sub}}, d^x, c, \eta$ gives the displayed derivative. Strict feasibility and $l > 0$ imply $R^{\text{time}} > 0$. If $a \le B^{\text{sub}}c$, both terms in l_w are nonpositive and the first is strictly negative. Rearranging $l_w > 0$ gives the stated threshold. The conclusion is local to the interior branch and does not extend its formula through a no-mining or other constraint switch. □

A.1.2 Consolidated Resources

Proof of Proposition 1.4. The household budget binds because retained cash has strictly positive marginal utility. Summing those budgets and using local-good clearing gives

$$\sum_v p_v \Big[F_v - (1-\alpha)C_v\Big] = \sum_{vw} t_{vw}(p_v - p_w/\delta_{vw}) = 0.$$

The first equality is physical clearing, with delivered imports divided by the iceberg factor; the second is shipment complementarity. Thus only mineral revenue, opening wealth, import expenditure and retained cash remain, which is (10). Substitute $p_v^z = c_v^z + \mu_z$ and $p_v^g = c_v^g + \mu_g$ into this identity and into (9). The two import margins cancel, yielding (11). Real transport losses remain in physical clearing; zero-profit shipment does not assert their absence. □

A.1.3 Regional Output as an Implemented Choice

Proof of Theorem 1.5. Fix an attainable positive output q. Every implementation costs at least $\mathcal{C}(q)$ and the attainable downstream stock value is $W(q)$, so its profit is no greater

than $W(q) - \mathcal{C}(q)$. An attained fiber minimizer and an attained downstream choice achieve this bound. Maximizing over the attainable outputs and comparing the separately specified exit payoff proves (13). A pure downstream policy changes neither the regional feasible graph nor $C^{\text{impl}}(z)$; hence it changes neither the fiber minimum nor $\Gamma^{\text{impl}}(q)$ at a fixed quantity. Multiple minimizers can have different quotes or household allocations, so unchanged output alone does not select unchanged nominal terms or a unique physical allocation. □

The resource identity also explains the imported-subsistence financing bound used in the existence discussion. On an active network with $d_A = \sum_{v \in \mathcal{V}^{\text{act}}} \alpha c_v^z c_v - \sum_{v \in \mathcal{V}^{\text{act}}} a_v > 0$, all remaining resource costs nonnegative, $C_v > c_v$ implies $C^{\text{impl}}(z) \geq d_A$. If $W(q) \leq \bar{v}q$ and operation is nonloss-making, then $d_A \leq C^{\text{impl}}(z) \leq W(q) \leq \bar{v}q$. This proves $q \geq d_A/\bar{v}$, without proving nonemptiness of the implementation graph or attainment of this lower bound. Sufficient compactness conditions, household optimality and the linked equilibrium construction remain in Online Appendix A.1.

B Proofs: Global Regulation and Production

These proofs establish the main results of Section 2 under their stated hypotheses. The institutional comparison is restricted to efficient complete outcomes with public retention value, independent private types, voluntary participation and balanced transfers. The subsequent resilience comparisons hold within the committed auction.

B.1 The Selling Institution and Its Conditional Optimality

B.1.1 Truthfulness and Critical Payments

Proofs of Proposition 2.1 and Lemma 2.5. Fix other reports, stock, capacity, retention and policy. Buyer i's utility after the pivot payment is true total welfare at the selected allocation minus $S_{n,-i}$, a quantity independent of its report. Truthful reporting maximizes that welfare over the unchanged feasible set. An allocation excluding i is feasible when i is present, so truthful utility $S_n - S_{n,-i}$ is nonnegative.

For critical payments, let $y_0 = q - n\bar{q}$. Telescoping the seller orders gives $H(y_0 + j\bar{q}) = H(y_0) + \sum_{u=1}^{j} \rho_u(y_0)$. Thus efficiency selects the n highest orders, respecting the fixed tie priority. The selected rival orders when i wins k_i^* packages occupy the first $n - k_i^*$ positions in its rival book. Removing the whole buyer fills the next k_i^* positions. The sum of those newly selected rival values is exactly the welfare difference in the pivot payment, giving (22). Removing only winning orders would incorrectly retain the same owner's losing orders in this counterfactual. □

B.1.2 The Efficient-Resale Seller-Value Bound

Proof of Theorem 2.3 and Corollary 2.4. Fix the stock and begin with $\tau^{\mathrm{pol}} = 0$. Any admissible complete-game Bayesian equilibrium induces a direct mechanism: simulate the entire strategy of each reported type, then return terminal allocations and cumulative transfers. A true type can imitate another type's entire strategy; independent types give the same law of rival information under such imitation. The direct outcome is therefore Bayesian incentive compatible, and the complete opt-out option makes it interim individually rational. Terminal efficiency and balanced transfers imply that seller cash plus true retention equals total efficient welfare minus total buyer utility.

To identify these utilities, fix a buyer type vector $\boldsymbol{b}_i$ and vary its report on the ray $r\boldsymbol{b}_i$, $0 \le r \le 1$. The compact convex type domain contains this ray and zero. Incentive compatibility implies that interim utility on the ray is convex, Lipschitz, and has derivative $\boldsymbol{b}_i \cdot \mathbb{E}_{-i} x_i^{\mathrm{ord}}(r\boldsymbol{b}_i, \boldsymbol{b}_{-i})$ almost everywhere, where x_i^{ord} is the vector indicating the marginal packages acquired. Expected efficient welfare has the same derivative: an efficient allocation supplies a subgradient of the welfare maximum. Integrating along the ray gives

$$U_i^M(\boldsymbol{b}_i) - U_i^M(0) = \mathbb{E}_{-i}\Big[S_Q(\boldsymbol{b}_i, \boldsymbol{b}_{-i}) - S_Q(0, \boldsymbol{b}_{-i})\Big].$$

At zero, returning all its packages to the seller removes zero buyer value and weakly raises retention, so $S_Q(0, \boldsymbol{b}_{-i}) = S_{Q,-i}(\boldsymbol{b}_{-i})$. The right side is exactly pivot utility $U_i^V(\boldsymbol{b}_i)$. Thus $U_i^M = U_i^V + c_i^{\mathrm{IR}}$ with $c_i^{\mathrm{IR}} = U_i^M(0) \geq 0$. The complete cash account now yields

$$\mathbb{E}R^M = W_Q(q, 0) - \sum_i c_i^{\mathrm{IR}} \leq W_Q(q, 0).$$

This is an efficient-outcome bound, not optimality over inefficient mechanisms or a privately informed seller's truth-telling theorem.

For attainment, use truthful pivot reports, the stated terminal consent rule and a seller strategy of passing. At truthful reports the initial allocation is efficient, so a balanced trade accepted by every buyer cannot strictly improve the seller. Against a unilateral buyer deviation, the other buyers' truthful reports and the pivot identity leave their combined initial value with the seller equal to $S_{Q,-i}$, independent of that buyer's report. Their initial payoffs are protected by consent and by the seller's option to pass. The deviator's final utility is at most total attainable true welfare minus that complement value, $S_Q - S_{Q,-i}$, its truthful utility. Initial nonparticipation gives the same bound because the initial auction then produces $S_{Q,-i}$. This constructs the asserted ex post Nash equilibrium; it makes no sequential-equilibrium claim at off-path histories. Genuine retention therefore attains the bound. Every alternative announcement with an admissible complete equilibrium is subject to the same bound, proving the corollary.

At a fixed public $\tau^{\mathrm{pol}} \geq 0$, apply the same argument to net final-holder values. The public compliance term does not change the derivative in the private type vector; a zero regulated type has nonpositive net values and can again be removed without lowering welfare. The conclusion retains final-holder incidence and the same admissible efficient complete-outcome class. Full game specifications are in Online Appendix A.2.2. □

B.2 Payment Floors and Exact Exposure

B.2.1 Reallocation and Retained Stock

Proofs of Propositions 2.6 and 2.8. At fixed stock and capacity, a higher charge lowers regulated orders and leaves every unaffected order and seller order unchanged. In the highest- orders allocation it can only improve the rank of an unaffected buyer's orders, under the same tie priority. The statewise inequality gives the claimed stochastic dominance under the unchanged type law. For laundering apply the partial and total chain rules to $L = \ell(q - \bar{q}m^{\mathrm{sold}})$, obtaining (27). At fixed stock an unchanged integer allocation leaves laundering unchanged. With random types, moving allocation thresholds can nevertheless change the probability of each integer allocation, so local statewise constancy alone does not justify differentiating an expectation as zero. □

B.2.2 Retention and Remaining-Market Floors

Proofs of Propositions 2.7 and 2.9. If winner i is removed, the available unselected seller orders include $\rho_1(y), \dots, \rho_{k_i^*}(y)$. The best replacement orders have respective values no smaller than these, so $c_{iu} \geq \rho_u(y)$ and $p_i \geq D_i(y)$. With $d_i = k_i^* \bar{q}$, concavity gives

$$H(q) - H(y) \leq \sum_i [H(y + d_i) - H(y)] \leq \sum_i p_i.$$

Adding actual retained value proves $R_n \geq H(q)$. In addition, every critical rank $n - k_i^* + u$ is at most n. Its rival book contains all unaffected orders except i's and all n seller orders. Its value is therefore at least the nth highest value of this sub-book, and at least the minimum of such values over buyer exclusions. Adding seller orders cannot reduce this floor. Combine the two bounds to obtain (29); comparison with $\bar{q}\theta_B$ gives the acquisition-margin statement, not a regional optimality condition. □

B.2.3 The Exact Exposure Identity

Proof of Theorem 2.10. The pivot transfers and retained value satisfy the exact identity

$$R_n = \sum_i S_{n,-i} - (N-1)S_n.$$

Each welfare term maximizes finitely many affine functions of τ^{pol}. It is continuous and piecewise affine; away from breakpoints its derivative is $-\bar{q}$ times its selected regulated quantity. Differentiation gives $\partial_{\tau^{\text{pol}}} R_n = -\bar{q}\chi_n$. Removing a winner preserves all previously selected rival orders and adds exactly its k_i^* replacement orders. The regulated orders among these replacements number between zero and k_i^*; summing over owners proves $0 \leq \chi_n \leq \sum_i k_i^* \leq n$. Thus the seller value is Lipschitz with constant $\bar{q}n$. Integrate over policy, including breakpoints, and apply Tonelli to the nonnegative bounded exposure integrand to obtain (32). The assumed integrability of positive buyer values and finite $H(q)$ ensure finite expected levels. The same exposure bound yields (33). Winning regulated quantity alone is not the integrand. □

B.3 Global Resilience and Local Production Transmission

B.3.1 The Global Profit Comparison and Its Corollaries

Proofs of Theorem 2.11 and Corollaries 2.12–2.13. Apply the integrated exposure identity separately to the baseline and each feasible competitor, at each choice's own stock and capacity, then subtract. The implementation costs have no direct policy term. This gives (35); requiring all such differences to be nonnegative is exactly global optimality. Strictness against every choice of different output makes the optimal physical quantity unique, without making nominal implementing terms unique.

If the baseline has zero exposure almost everywhere, it loses no receipts. Every competitor's receipts weakly decrease, so

$$\Pi(\Xi, \tau^{\mathrm{pol}}) \leq \Pi(\Xi, 0) \leq \Pi(\Xi_0, 0) = \Pi(\Xi_0, \tau^{\mathrm{pol}}).$$

Only baseline maximizers can remain tied. This proves the protection and common-baseline-output conclusions. If all choices in $\mathcal{X}^{\mathrm{eq}}$ have common exposure $\kappa(t)$, their common loss cancels from every comparison inside $\mathcal{X}^{\mathrm{eq}}$. The assumed comparisons outside $\mathcal{X}^{\mathrm{eq}}$ then retain the unique baseline. With an outside gap $\delta^{\mathrm{gap}} > 0$ and at most $\bar{Q}$ packages, differential policy loss is no greater than $\bar{q}\bar{Q}\tau^{\mathrm{pol}}$. The strict sufficient interval follows by comparing this bound with δ^{gap}. No uniform gap is inferred for arbitrary nearby outputs in a continuous choice set. □

B.3.2 The Local Derivative and Primitive Attenuation

Proofs of Theorem 2.14 and Corollary 2.15. On the stipulated fixed smooth feasible regime, differentiate the regional first-order conditions: $\mathcal{H} z_{\tau^{\mathrm{pol}}} + W_{q\tau^{\mathrm{pol}}} \boldsymbol{g}_q = 0$, with $\boldsymbol{g}_q = \nabla_z q$. Multiplication by $\boldsymbol{g}_q{}^{\mathsf{T}}$ gives (37). Negative definiteness of $\mathcal{H}$ and $\boldsymbol{g}_q \neq 0$ imply $\Lambda = -\boldsymbol{g}_q{}^{\mathsf{T}} \mathcal{H}^{-1} \boldsymbol{g}_q > 0$. Differentiability of expected exposure gives $W_{q\tau^{\mathrm{pol}}} = -\bar{q}\partial_q \bar{\chi}_n = -\gamma$ and therefore $q^*_{\tau^{\mathrm{pol}}} = -\Lambda\gamma$. At the same zero-policy optimum a direct levy subtracts $tq(z)$, giving the same Hessian and policy-gradient term $-\boldsymbol{g}_q$; its output derivative is $-\Lambda$.

For one package, assume F_U is continuous at the relevant reserve values. Except on zero-probability ties, a regulated order sets the critical payment precisely when $V_U > V_R - \bar{q}\tau^{\mathrm{pol}} > \rho(q)$. Independence therefore gives

$$\bar{\chi}_1(q, \tau^{\mathrm{pol}}) = \int_{\rho(q)+\bar{q}\tau^{\mathrm{pol}}}^{\infty} [1 - F_U(r - \bar{q}\tau^{\mathrm{pol}})] f_R(r)\, dr.$$

Differentiating its lower endpoint in q and multiplying by $\bar{q}$ gives (39). Concavity makes $\rho' \leq 0$. The strict reserve, probability, density and product conditions yield $0 < \gamma < 1$, establishing strictly partial contraction on the maintained regular branch, not at a package-count or active-set switch. □

B.4 The Global Policy Ceiling

Proof of Theorem 2.16. At fixed stock, capacity and realized types, once the common compliance bill exceeds every regulated marginal value, regulated orders can be replaced by nonnegative retention without lowering welfare in either the full allocation or any buyer-exclusion problem. Let the common resulting welfare be the unaffected-only welfare. In $R_n = \sum_i S_{n,-i} - (N-1)S_n$, each absent regulated buyer contributes one copy of the full welfare, cancelling its copy in $(N-1)S_n$. The seller value is therefore $R_n^{\mathcal{U}}$. Since the

exact exposure identity makes receipts weakly decreasing, $R_n \geq R_n^{\mathcal{U}}$ at every finite charge. Moreover,

$$H(q) \leq R_n \leq H(q) + \sum_{i,k} (b_i^k)_+.$$

The upper bound follows from individual rationality and $H(y) \leq H(q)$; the assumed integrability permits dominated convergence. Hence $W_n(q, \tau^{\mathrm{pol}}) \downarrow \underline{W}_n(q)$, proving the receipt floor and the stated exact supremum of the fixed-stock receipt loss. A finite uniform upper bound on regulated values gives one finite exclusion charge for every feasible stock and capacity, and thus equality of the optimized profit and its remaining-market floor beyond that charge.

The floor value at each implementation is a feasible lower bound on its policy payoff. Taking maxima and including the zero-profit exit option therefore gives $\Pi^*(\tau^{\mathrm{pol}}) \geq \underline{\Pi} > 0$, which excludes exit. For every selected active optimum, the uniform implementation-cost and stock-value bounds imply

$$\underline{\Pi} \leq W(q_{\tau^{\mathrm{pol}}}, \tau^{\mathrm{pol}}) - \mathcal{C}(q_{\tau^{\mathrm{pol}}}) \leq \bar{v} q_{\tau^{\mathrm{pol}}} - c_{\min}^{\mathrm{act}}.$$

Rearranging proves both inequalities in (43). The same argument applies at zero policy, so the proportional-contraction bound is nonnegative and strictly less than one. These are uniform bounds under the stated positive-surplus and positive-cost hypotheses; a mere positive per-package acquisition margin does not establish them. □

C Proofs: Blockchain as a Certification Device

This appendix proves the main results of Section 3 under its maintained physical, contractual and funding assumptions. Further channel, aggregation and resource-cost results are in Online Appendix C.1.1–C.1.11.

C.1 Conservation and Retrospective Exclusion

Proof of Proposition 3.1. Induct on the ordered events. A mint adds measured backing disjoint from all surviving claims. A transfer replaces its inputs with disjoint portions of that backing and records the residual as loss. Redemption removes backing from the live claim account. These operations preserve unique backing and give (46) by additivity and telescoping. If previously redeemed material is validly readmitted, its mass enters A_t again, matching its earlier exit in X_t; the identity is a live-stock account, not a count of distinct historical extraction. Fresh identifiers and previously live inputs preserve an acyclic finite history. Status events alter eligibility only.

At an effective revocation, every existing claim containing that origin in its ancestry is excluded by (45). Every successor inherits that ancestry. Induction therefore excludes all later recorded descendants at the status check governing their settlement. The ordered validation rule prevents a concurrently submitted transfer from using a stale status. Intake checks prevent the same excluded physical origin from escaping through another mint. No subsequent event changes this conclusion without the specified reinstatement decision. □

C.2 Investigation and Voluntary Disclosure

Proof of Proposition 3.2. Given conclusive evidence, reporting costs $L_{E,j}$; concealment has expected cost $P^{\text{disc}}{}_{-j}L_{L,j} + (1 - P^{\text{disc}}{}_{-j})L_{U,j}$. Subtraction gives (47). Before investigation, these alternatives differ only when the claim is defective and early evidence is produced. Multiplying by that probability and subtracting effort cost yields (48). Continuity on the compact feasible effort interval gives a maximizer. Concavity of s_j, positivity of $A^{\text{disc}}{}_j$ and strict convexity of c_j make that objective strictly concave, establishing uniqueness. Its first derivative at an interior optimum gives (49). □

C.3 Funded Capture with Optional Suppression

Proof of Theorem 3.3. Fix an action a^{div} and silence set $\mathcal{T}^{\text{sil}}$. An original recipient $i \neq 0$ paid gross transfer t_i gains $\lambda_i t_i - a_i(\mathcal{T}^{\text{sil}})$ relative to honesty. Acceptance therefore requires $t_i \geq a_i(\mathcal{T}^{\text{sil}})/\lambda_i$. Each purchased reporter analogously requires $t_j \geq \sigma_j(\mathcal{T}^{\text{sil}})/\lambda_j$. The joint implementability assumption makes these bounds sufficient as well as necessary. Thus every accepted schedule implementing this silence set costs at least $\mathcal{B}^{\text{adv}}(\mathcal{T}^{\text{sil}})$ before settlement.

If that bill exceeds $F_0^{\mathrm{adv}}(a^{\mathrm{div}}, \Omega)$, no such schedule is funded. Otherwise equality payments are funded and leave organizer surplus

$$G_\Omega(a^{\mathrm{div}}) - a_0(\mathcal{T}^{\mathrm{sil}}) - \mathcal{B}^{\mathrm{adv}}(\mathcal{T}^{\mathrm{sil}});$$

larger transfers cannot improve it. Since $\mathcal{J}^{\mathrm{rep}}$ is finite, the minimum over nonempty funded choices is attained. Maximizing organizer surplus is therefore exactly subtracting $K_J^F(a^{\mathrm{div}})$ from $G_\Omega(a^{\mathrm{div}})$. The empty case has no funded implementation. Applying the same comparison to every joint action proves both deterrence statements. At equality the organizer and recipients can be indifferent, so excluding indifferent implementations requires the strict inequality or a specified honest tie rule. □

C.4 Positive-Trade Interior Equilibrium

Proof of Theorem 3.4. After learning with certainty that its unit is fraudulent, the buyer loses L_E by reporting and expects $r^{\mathrm{late}}L_L$ by remaining silent. It therefore requires a delivered payment of at least A^{disc} to accept silence. The entire original coalition together can save at most $(1 - r^{\mathrm{late}})D$: reporting forfeits its bonds with certainty, whereas silence leaves expected forfeitures $r^{\mathrm{late}}D$. By (55), its maximum willingness to pay is smaller than the buyer's reservation payment. Thus no silence agreement can leave all its participants weakly better off, even with coalition-wide cofinancing. Bilateral willingness to pay cannot exceed this coalition bound. Earlier mint bribes are sunk at this subgame; reshuffling them creates no extra suppression surplus.

Strict convexity in (56) gives the buyer's unique best response. For an interior solution the uniform cost distribution gives

$$\bar{c^{\mathrm{forg}}}\pi = \Gamma^{\mathrm{adm}} - r^{\mathrm{late}}D - D(1 - r^{\mathrm{late}})\alpha^{\mathrm{scr}}\pi,$$

yielding (58). The fraud fraction lies strictly inside the unit interval because its numerator is positive and less than $\bar{c^{\mathrm{forg}}}$; the last inequality in (57) ensures $e^* = \alpha^{\mathrm{scr}}\pi^* < 1$. More generally, the fraud best response is the continuous, nonincreasing map

$$\pi \longmapsto \left[\frac{\Gamma^{\mathrm{adm}} - DP^{\mathrm{disc}}(\min\{\alpha^{\mathrm{scr}}\pi, 1\})}{\bar{c^{\mathrm{forg}}}}\right]_0^1,$$

where brackets truncate to $[0, 1]$. Its difference from π is strictly decreasing; hence another fixed point is impossible. Each signer is indifferent between its funded offer and its maintained honest payoff, and $P^{\mathrm{disc}}(e)\sum_i b_i \le \sum_i b_i \le F_{\mathrm{adv}}$. The opening bargaining comparison establishes sequential reporting and the absence of mutually acceptable silence contracts. Posted wages, prepaid fees and segregated deposits satisfy the stated participation and budget contracts; buyer participation completes the positive-trade allocation.

For the matched benchmark, $\pi_0 = \widetilde{\Gamma^{\mathrm{adm}}}/\bar{c^{\mathrm{forg}}}$ and $e_0 = 0$. Equation (58) gives $\pi^* < \pi_0$ and $P^{\mathrm{disc}*} > r^{\mathrm{late}}$. The live-contamination function in (60) has derivatives $(1 - P^{\mathrm{disc}})/(1 -$

$\pi P^{\text{disc}})^2 > 0$ with respect to π and $-\pi(1-\pi)/(1-\pi P^{\text{disc}})^2 < 0$ with respect to P^{disc} in this interior class. Therefore $\iota(\pi^*, P^{\text{disc}^*}) < \iota(\pi_0, r^{\text{late}})$, accounting for the smaller surviving denominator as well as fewer illegal admissions. □

D Proofs: Global Effects of Certification on Prices and Directed Innovation

These essential implications support Section 4. Full derivations, admitted parameter cases and surviving-vintage refinements remain in Online Appendix K. The certification-date North–South comparison and CES growth application remain distinct demand specifications; these proofs do not establish a single transition between them.

D.1 What Certification Preserves

In the matched certification-date market, hold available ordinary labor, nonfocal technologies, active networks, assignment and ownership fixed, with the uniform certified charge the only changed receipt wedge. Certified patents survive until displacement; dirty prototype gains last one interval in both regimes. On the admissible machine-margin branch, the certified inventor's first-flow multiplier is

$$M_C = \frac{(1-\phi_{NS})E_C^C}{E_C^P}.$$

A weak increase in net certified spending weakly raises this first flow and weakly lowers the mobile proportional owned dirty prototype gain and the fixed-capacity coerced operating gain. These statements concern operating flows, not complete owner rewards.

For $x \in \{P, C\}$ use a common positive monetary normalization c^{norm}, finite, consistently valued complete rewards $\widehat{J}_i^x = J_i^x / c^{\text{norm}}$, effort $s_x = (S^{\text{sci}} - S^{\text{ver}\,x})/N_C > 0$, and differentiable, nondecreasing convex normalized laboratory costs $R_i^{\text{lab}\,x}$, with $R_i^{\text{lab}\,x}(0) = 0$. Retain the current scientist-market and funding hypotheses. With all rival research payments zero, define

$$\mathcal{R}_x^{\text{alt}} = \max\{0, \max_{i \neq C}(h_i'(0)[\widehat{J}_i^x]_+ - R_i^{\text{lab}\,x\prime}(0))\},$$

$$\Lambda_x = \frac{R_C^{\text{lab}\,x\prime}(s_x) + \mathcal{R}_x^{\text{alt}}}{h_C'(s_x)}, \qquad \Gamma_x = \Lambda_x - \widehat{J}_C^x.$$

Write Δ for certification minus pooling. Let ΔO_C be the complete discounted operating-value change, including survival-weighted later rents, and $\Delta \mathcal{C}_C$ the complete dated contractual-cost change, with refunds and no duplicate deductions.

Proposition D.1 (Exact Preservation of a Private Pooled Allocation)**.** *A strictly private pooled allocation of margin $m_P = -\Gamma_P > 0$ remains strictly private exactly when*

$$\Delta \mathcal{C}_C < \Delta O_C - c^{\text{norm}} \Delta\Lambda + c^{\text{norm}} m_P.$$

Equality gives a research tie. The stronger condition $\Delta \mathcal{C}_C \leq \Delta O_C - c^{\text{norm}}\Delta\Lambda$ weakly lowers the untruncated support threshold. Neither condition compares two complete dynamic equilibrium paths.

Proof. The exact research criterion is $\Gamma_x < 0$ for strict private dominance, and $[\Gamma_x]_+$ for minimum weak support. Subtracting the definitions and using $\Delta J_C = \Delta O_C - \Delta \mathcal{C}_C$ gives $\Gamma_C = -m_P + \Delta\Lambda - (\Delta O_C - \Delta \mathcal{C}_C)/c^{\text{norm}}$. The two assertions follow by rearrangement. The spending identity and operating-flow comparison are proved in Online Appendix K.1–K.3. □

D.2 A Sharper Sufficient Withdrawal Threshold

Retain the two-type, quadratic-cost and invariant-domain hypotheses of Theorem 4.6, with complete unsubsidized rewards satisfying its envelopes, no rival grants, and $m_\infty = H_C j_0 - r_C s^* > 0$. Put

$$\zeta_\Sigma = 1 - \theta_\Sigma \in (0,1), \qquad \mathfrak{a} = H_C K_C + \nu_D^R D_0, \qquad \mathfrak{b} = \nu_D^R D_1,$$
$$f(A) = \mathfrak{a}/A + \mathfrak{b}/A^{\zeta_\Sigma}.$$

Proposition D.2 (Refined Private-Continuation Certificate)**.** *The private marginal advantage is at least $m_\infty - f(A)$ and $G_C^{\min} \le [f(A) - m_\infty]_+/H_C$. Let ϱ solve $f(\varrho) = 3m_\infty/4$ when $\mathfrak{a} + \mathfrak{b} > 0$, and set $\varrho = 0$ otherwise. Then $A^\dagger = \max\{A_{\min}, \varrho\} \le A^*$, with A^* from (73). At $A \ge A^\dagger$, the private marginal advantage is at least $m_\infty/4 > 0$. If technology does not decrease and the selected unsubsidized continuation preserves the stated domain, support can cease permanently there.*

Proof. Subtract the dirty upper envelope from the certified lower envelope. The exact support identity divides the negative of this advantage by $H_C > 0$. If a coefficient is positive, f is continuous and strictly decreasing from infinity to zero, so the root is unique. At $A = A^*$, each of $H_C K_C/A$, $\nu_D^R D_0/A$, and $\nu_D^R D_1/A^{\zeta_\Sigma}$ is at most $m_\infty/4$; hence $f(A^*) \le 3m_\infty/4$ and $A^\dagger \le A^*$. The zero-coefficient case is immediate. Persistence follows from the maintained domain and nondecreasing technology. □

Under the exact cohort growth law, replace A^* by $A^\dagger$ in the ceiling formula for the funded withdrawal date. Equality at the cutoff retains a strict margin. This remains a sufficient certificate; failing it does not establish lock-in. Online Appendix K.3 gives closed forms, stochastic hitting and the actual-limit classification.

D.3 Mine Receipts and Protection Payments

On the common-quote installed-input coerced branch, with fixed positive capacity and procurement coefficient and $m_\beta = 1/\beta_m > 1$, $Q_r(p) = z_r p^{m_\beta - 1}$ and $\Pi_r(p) = z_r p^{m_\beta}/m_\beta$, where $z_r > 0$ is fixed. Thus

$$\frac{\Pi^C}{\Pi^P} = \frac{F_0}{F(p_C/p_D)}, \qquad \frac{Q^C}{Q^P} = \left(\frac{F_0}{F(p_C/p_D)}\right)^{(m_\beta - 1)/m_\beta}.$$

These identities follow directly from snapshot clearing $(p_D/p_0)^{m_\beta} = F_0/F(p_C/p_D)$. Consequently the current price threshold lowers this mine's output, profit and proportional gross prototype gain exactly when $\nu_{NS} > \nu_{NS}^{-}$. This does not sign actual protection payments: the model supplies no fixed-share payment rule. Its growth result instead bounds their ratio to A_C by $B_0^{\text{prot}}/A_C + B_1^{\text{prot}}/A_C^{1-\theta_\Sigma} \to 0$, under the stated dirty-sales affordability bound. New local payments end at the production date after successful control; historical claims remain. See Online Appendix K.20 and K.4.2.

E Proofs: Building State Capacity after Certification

This appendix gives the exact funded intervention extension supporting Section 5. Full changing-path and balance-dependent arguments remain in Online Appendix K.6.

E.1 An Exact Funded Intervention Threshold

Let $d^G = \delta_G B_{t+1}/B_t \in [0,1)$ be constant. Retain the stationary reduced Bellman problem, with $b \geq 0$, a nonempty compact feasible tax set, continuous attempt costs $k(\tau) \geq 0$, success probabilities $P(\tau) \in (0,1]$, and legal control values $u(\tau)$ measured at the control date. All compared plans, including repeated failures, must be funded; waiting and failure return to the same normalized uncontrolled state, without a binding balance distinction.

Proposition E.1 (Tax-Dependent Success and Paid Catch-Up)**.** *Immediate intervention, repeated after failure at a maximizing tax, is strictly preferable to waiting exactly when*

$$b < b^{\text{crit}}, \qquad b^{\text{crit}} = \max_{\tau \in \mathcal{T}^{\text{tax}}} \frac{(1-d^G)[d^G P(\tau)u(\tau) - k(\tau)]}{1 - d^G + d^G P(\tau)}.$$

At equality, waiting and at least one maximizing attempt tie. If the funded legal plan has normalized public flows $r^G_{a^{\text{age}}}(\tau)$ for ages $a^{\text{age}} = 0, \ldots, H-1$ and stationary mature flow $r^G_\infty(\tau)$, then

$$u(\tau) = \sum_{a^{\text{age}}=0}^{H-1} {d^G}^{a^{\text{age}}} r^G_{a^{\text{age}}}(\tau) + \frac{{d^G}^H r^G_\infty(\tau)}{1-d^G}.$$

Proof. The waiting action's fixed point is $b/(1-d^G)$; the repeated-attempt fixed point is $[d^G P(\tau)u(\tau) - k(\tau)]/[1 - d^G + d^G P(\tau)]$. The maximum of these fixed points solves the Bellman equation: every action lies weakly below it and a maximizing action attains it. All action slopes are bounded by $d^G < 1$, ensuring uniqueness. Comparing the fixed points gives the threshold. Discounting the finite catch-up flows and summing the mature geometric tail gives the control value. □

These public flows include support and services; parent-paid adoption costs enter private participation and are not subtracted twice. Changing paths retain the future uncontrolled value in the intervention test, and binding finance requires the balance-dependent Bellman equation. The formula is not a primitive-only technology cutoff. See Online Appendix K.6 for the complete statement and proof.